### Section 9 – 4A1: Logarithmic Functions

#### Finding the y Intercept for an **increasing** logarithmic function

To find the x intercept let $y = 0$

Get the log expression alone.

Convert the log expression to an exponential expression. Solve for $x$

Find the x intercept for $y = \log_2 x$

- if $y = 0$
  - $0 = \log_2 x$
  - $2^0 = x$
  - $1 = x$

$(1,0)$

**Example 1**

Find the x intercept for $y = \log_2 (x - 3)$

- if $y = 0$
  - $0 = \log_2 (x - 3)$
  - $2^0 = x - 3$
  - $1 + 3 = x$
  - $4 = x$

$(4,0)$

**Example 2**

Find the x intercept for $y = \log_4 (x + 6)$

- if $y = 0$
  - $0 = \log_4 (x + 6)$
  - $4^0 = x + 6$
  - $1 - 6 = x$
  - $-5 = x$

$(-5,0)$

**Example 3**

Find the x intercept for $y = \log_2 (x) - 3$

- if $y = 0$
  - $0 = \log_2 (x) - 3$
  - $3 = \log_2 (x)$
  - $2^3 = x$
  - $8 = x$

$(8,0)$

**Example 4**

Find the x intercept for $y = \log_3 (x) + 2$

- if $y = 0$
  - $0 = \log_3 (x) + 2$
  - $-2 = \log_3 (x)$
  - $3^{-2} = x$
  - $1/9 = x$

$(1/9,0)$

**Example 5**

Find the x intercept for $y = \log_5 (-x)$

- if $y = 0$
  - $0 = \log_5 (-x)$
  - $5^0 = \log_3 (-x)$
  - $1 = -x$
  - $-1 = x$

$(-1,0)$

**Example 6**

Find the x intercept for $y = -\log_2 (x)$

- if $y = 0$
  - $0 = -\log_2 (x)$
  - $0 = \log_3 (x)$
  - $3^0 = x$
  - $1 = x$

$(1,0)$

---

#### Finding the y Intercept for a **decreasing** exponential function
To find the x intercept let \( y = 0 \)
Get the log expression alone.
Convert the log expression to an exponential expression. Solve for \( x \)

Find the y intercept for \( y = (\frac{1}{2})^x \)

if \( x = 0 \)

\( y = (\frac{1}{2})^0 \)
\( y = 1 \)

\((0,1)\)

Example 7
Find the x intercept for \( y = \log_{1\frac{1}{2}}(x - 4) \)

if \( y = 0 \)

\( 0 = \log_{1\frac{1}{2}}(x - 4) \)
\( (1\frac{1}{2})^0 = x - 4 \)
\( 1 + 4 = x \)
\( 5 = x \)

\((5,0)\)

Example 9
Find the x intercept for \( y = \log_{1\frac{1}{2}}(x + 3) \)

if \( y = 0 \)

\( 0 = \log_{1\frac{1}{2}}(x + 3) \)
\( (1\frac{1}{2})^0 = x + 3 \)
\( 1 - 3 = x \)
\( -2 = x \)

\((-2,0)\)

Example 8
Find the x intercept for \( y = \log_{1\frac{1}{2}}(x - 4) \)

if \( y = 0 \)

\( 0 = \log_{1\frac{1}{2}}(x - 4) \)
\( (1\frac{1}{2})^0 = x - 4 \)
\( 1 + 4 = x \)
\( 5 = x \)

\((5,0)\)

Example 10
Find the x intercept for \( y = \log_{1\frac{1}{2}}(x) - 4 \)

if \( y = 0 \)

\( 0 = \log_{1\frac{1}{2}}(x) - 4 \)
\( 4 = \log_{1\frac{1}{2}}(x) \)
\( (1\frac{1}{2})^4 = x \)
\( 1/81 = x \)

\((1/81,0)\)

Example 11
Find the x intercept for \( y = \log_{1\frac{1}{2}}(-x) \)

if \( y = 0 \)

\( 0 = \log_{1\frac{1}{2}}(-x) \)
\( (1\frac{1}{2})^0 = -x \)
\( 1 = -x \)
\( -1 = x \)

\((-1,0)\)

Example 12
Find the x intercept for \( y = -\log_{1\frac{1}{2}}(x) \)

if \( y = 0 \)

\( 0 = -\log_{1\frac{1}{2}}(x) \)
\( -0 = \log_{1\frac{1}{2}}(x) \)
\( (1\frac{1}{2})^0 = x \)
\( 1 = x \)

\((1,0)\)

Finding the value for \( y \) given a value for \( x \)
for a decreasing exponential function

\[ f(3) \text{ means that } x = 3 \]

\[ f(x) \text{ can be replaced with the variable } y \]

<table>
<thead>
<tr>
<th>Example 13</th>
<th>Example 14</th>
<th>Example 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \log_3(x - 2) )</td>
<td>( f(x) = \log_2(x + 6) )</td>
<td>( f(x) = \log_2(x) + 4 )</td>
</tr>
<tr>
<td>( y = \log_3(11 - 2) )</td>
<td>( y = \log_3(10 + 6) )</td>
<td>( y = \log_2(8) + 4 )</td>
</tr>
<tr>
<td>( y = \log_3(9) )</td>
<td>( y = \log_2(16) )</td>
<td>( y = 3 + 4 )</td>
</tr>
<tr>
<td>( y = \log_3(9) )</td>
<td>( y = \log_2(16) )</td>
<td>( y = 7 )</td>
</tr>
<tr>
<td>( y = 2 )</td>
<td>( y = 4 )</td>
<td>( y = 7 )</td>
</tr>
<tr>
<td>((11,2))</td>
<td>((10,4))</td>
<td>((8,7))</td>
</tr>
</tbody>
</table>

**Finding the value for \( y \) given a value for \( x \)**
for a *decreasing* exponential function

\[ f \left( \frac{1}{3} \right) \text{ means that } x = \frac{1}{3} \]

\[ f \left( \frac{1}{3} \right) \text{ can be replaced with the variable } y \]

**Example 19**

\[ f(x) = \log_{1/2}(x - 2) \]

find \( f(10) \)

\[ y = \log_{1/2}(10 - 2) \]

\[ y = \log_{1/2}(8) \]

\[ \text{[to find } \log_{1/2}(8) \text{ ask what power of } 1/2 \text{ is } 8] \]

\[ \log_{1/2}(8) = -3 \]

\[ y = -3 \]

\[ (10, -3) \]

**Example 20**

\[ f(x) = \log_{1/3}(x + 5) \]

find \( f(-4) \)

\[ y = \log_{1/3}(-4 + 5) \]

\[ y = \log_{1/3}(1) \]

\[ \text{[to find } \log_{1/3}(1) \text{ ask what power of } 1/3 \text{ is } 1] \]

\[ \log_{1/3}(1) = 0 \]

\[ y = 0 \]

\[ (-4, 0) \]

**Example 21**

\[ f(x) = \log_{1/2}(x) + 1 \]

find \( f(1/8) \)

\[ y = \log_{1/2}(1/8) + 1 \]

\[ \text{[to find } \log_{1/2}(1/8) \text{ ask what power of } 1/2 \text{ is } 1/8] \]

\[ \log_{1/2}(1/8) = 3 \]

\[ y = 3 + 1 \]

\[ y = 4 \]

\[ (1/8, 4) \]

**Example 22**

\[ f(x) = \log_{1/4}(x) - 3 \]

find \( f(16) \)

\[ y = \log_{1/4}(16) - 3 \]

\[ \text{[to find } \log_{1/4}(16) \text{ ask what power of } 1/4 \text{ is } 16] \]

\[ \log_{1/4}(16) = -2 \]

\[ y = -2 - 3 \]

\[ y = -5 \]

\[ (16, -5) \]

**Example 23**

\[ f(x) = \log_{1/5}(-x) \]

find \( f(-25) \)

\[ y = \log_{1/5}(-(-25)) \]

\[ y = \log_{1/5}(25) \]

\[ \text{[to find } \log_{1/5}(25) \text{ ask what power of } 1/5 \text{ is } 25] \]

\[ \log_{1/5}(25) = -2 \]

\[ y = -2 \]

\[ (-25, -2) \]

**Example 24**

\[ f(x) = -\log_{1/3}(x) \]

find \( f(81) \)

\[ y = -\log_{1/3}(81) \]

\[ \text{[to find } \log_{1/3}(81) \text{ ask what power of } 1/3 \text{ is } 81] \]

\[ \log_{1/3}(81) = -4 \]

\[ y = -(-4) \]

\[ y = 4 \]

\[ (81, 4) \]