Section 4 – 4: Greatest Common Factor (GCF)

Factoring
The last chapter introduced the distributive process. The distributive process takes a product of a monomial and a polynomial and changes the multiplication problem into a polynomial (the addition or subtraction of several terms).

The distributive process changes
the product $2(3x + 5)$ into the polynomial
$6x + 10$

The distributive process changes
the product $4x(2x + 3)$ into the polynomial
$8x^2 + 12x$

The last chapter also introduced the FOIL process. The FOIL process is a multiplication problem. The FOIL process takes the product of two binomials and changes the multiplication problem into a polynomial.

The FOIL process changes
the product $(x + 1)(x + 5)$ into the polynomial
$x^2 + 6x + 5$

The FOIL process changes
the product $(x + 2)(x - 2)$ into the polynomial
$x^2 - 4$

Factoring
Factoring is the process of changing a polynomial into either a distributive problem or a FOIL problem. Factoring reverses the distributive or FOIL process.

If the distributive process changes
$2(3x + 5)$ into $6x + 10$
then the factoring process reverses this and changes
$6x + 10$ into $2(3x + 5)$

If the FOIL process changes
$(x - 2)(x + 5)$ into $x^2 + 3x - 10$
then the factoring process reverses this and changes
$x^2 + 3x - 10$ into $(x - 2)(x + 5)$

There are three types of factoring that will be covered in this chapter.

1. Factoring out the Greatest Common Factor (GCF).
2. Factoring the Difference Of Two Perfect Squares.
3. Factoring Trinomials.
Factors

The factors of a whole number are all the whole numbers that divide evenly into that number.

The factors of 4 are 1, 2 and 4
The factors of 12 are 1, 2, 3, 4, 6 and 12
The factors of 5 are 1 and 5

Greatest Common Factor (GCF)

The Greatest Common Factor (GCF) for a group of whole numbers is the largest whole number that will divide evenly into every number in the group of numbers.

Example 1
6
is the largest number that will divide into
12 and 30
so the GCF is 6

Example 2
12
is the largest number that will divide into
12 and 24
so the GCF is 12

Example 3
1
is the largest number that will divide into
3 and 7
so the GCF is 1

Example 4
2
is the largest number that will divide into
6, 12 and 20
so the GCF is 2

Example 5
5
is the largest number that will divide into
5, 10 and 30
so the GCF is 5

Example 6
1
is the largest number that will divide into
2, 10 and 11
so the GCF is 1

Problems: Find the Greatest Common Factor (GCF) for each group of numbers:

1. 8 and 20
2. 6 and 15
3. 6 and 12
4. 3 and 10
5. 8, 16 and 32
6. 14, 21 and 28
7. 3, 9 and 18
8. 5, 8 and 12
9. 8, 12 and 28

Answers:
1. GCF is 4
2. GCF is 3
3. GCF is 6
4. GCF is 1
5. GCF is 8
6. GCF is 7
7. GCF is 3
8. GCF is 1
9. GCF is 4
Greatest Common Factor for Variable Terms

The Greatest Common Factor (GCF) can also be found for a list of variable terms.

If the list of variable terms has a common variable
then the GCF is the common variable
to the lowest power that the common variable has in the group of terms.

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$ and $x^6$</td>
<td>$y^3$ and $y^4$</td>
<td>$x$ and $x^3$</td>
</tr>
<tr>
<td>have a common variable of $x$</td>
<td>have a common variable of $y$</td>
<td>have a common variable of $x$</td>
</tr>
<tr>
<td>$x^2$ is the lowest power of $x$ in the two terms so $x^2$ is the GCF</td>
<td>$y^3$ is the lowest power of $y$ in the two terms so $y^3$ is the GCF</td>
<td>$x$ is the lowest power of $x$ in the two terms so $x$ is the GCF</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Example 4</th>
<th>Example 5</th>
<th>Example 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^4 \cdot x^2$, and $x^5$</td>
<td>$y^3$, $y^4$ and $y$</td>
<td>$x^3 y^3$, $xy^2$ and $y^5$</td>
</tr>
<tr>
<td>have a common variable of $x$</td>
<td>have a common variable of $y$</td>
<td>have a common variable of $y$</td>
</tr>
<tr>
<td>$x^2$ is the lowest power of $x$ in the three terms so $x^2$ is the GCF</td>
<td>$y$ is the lowest power of $y$ in the three terms so $y$ is the GCF</td>
<td>$y^2$ is the lowest power of $y$ in the three terms so $y^2$ is the GCF</td>
</tr>
</tbody>
</table>

Problems: Find the Greatest Common Factor for each list of variables:

1. The GCF of $x^2$ and $x^3$ is ______   2. The GCF of $x^4$ and $x^5$ is ______

3. The GCF of $y^3$ and $y^6$ is ______   4. The GCF of $y^4$, $y^3$ and $y^5$ is ______

5. The GCF of $x^2$, $x^3$ and $x^4$ is ______   6. The GCF of $x$, $x^2$ and $x^3$ is ______

7. The GCF of $x^4$, $x^3 y$ and $xy$ is ______   8. The GCF of $x^4$ and $y^4$ is ______

Answers:

1. GCF is $x^2$   2. GCF is $x^4$   3. GCF is $y^3$   4. GCF is $y^3$
5. GCF is $x^2$   6. GCF is $x$   7. GCF is $x$   8. no common variable
Finding the Greatest Common Factor (GCF) for a Polynomial

Each term in a polynomial may have both a number coefficient and one or more variables. To find the GCF for a polynomial find the GCF for all the coefficients and then find the GCF for the variables in each term and then multiply the GCF’s together.

Finding the Greatest Common Factor

1. Find the GCF for the coefficients (number in front a variable) of all the terms.
2. Find the GCF for each variable term by using the lowest power that it has in the group of terms.
3. The GCF is the product of steps 1 and 2.

Example 1:
Find the GCF of \(8x^2 + 12x\)
1. The GCF of 8 and 12 is 4
2. The GCF of \(x^2\) and \(x\) is \(x\)
The GCF is \(4x\)

Example 2:
Find the GCF of \(10y^4 + 15y^3\)
1. The GCF of 10 and 15 is 5
2. The GCF of \(y^4\) and \(y^3\) is \(y^3\)
The GCF is \(5y^3\)

Example 3:
Find the GCF of \(12x^4 + 9x^3 + 6x^2\)
1. The GCF of 12, 9, and 6 is 3
2. The GCF of \(x^4\), \(x^3\), \(x^2\) is \(x^2\)
The GCF is \(3x^2\)

Example 4:
Find the GCF of \(4y^5 + 8y^4 + 10y^3\)
1. The GCF of 4, 8, and 10 is 2
2. The GCF of \(y^5\), \(y^4\), \(y^3\) is \(y^3\)
The GCF is \(2y^3\)

Example 5:
Find the GCF of \(3y^5 + 8y^4 + 12y^3\)
1. The GCF of 3, 8, and 12 is 1
2. The GCF of \(y^5\), \(y^4\), \(y^3\) is \(y^3\)
The GCF is \(y^3\)

Example 6:
Find the GCF of \(16x^3 + 12x^2 + 8\)
1. The GCF of 16, 12, and 8 is 4
2. There is no common variable so the GCF is 1
The GCF is 4
Factoring out the GCF from a Polynomial

Factoring out a GCF from a polynomial is the reverse process of the distributive operation. The distributive operation multiplied a term outside a parentheses into each term of an expression inside the parentheses as seen in the following examples:

\[
\begin{align*}
2(5x + 3) &= 10x + 6 \\
5x(2x - 4) &= 10x^2 - 20x \\
3x(2x^2 - 5x) &= 6x^3 - 15x^2
\end{align*}
\]

Factoring out the GCF involves DIVIDING OUT THE GCF and placing it outside the parentheses as a product with the results of the division left inside the parentheses.

Example 1. Factor the GCF out of \(8x + 16\)

\(8x + 16\) has a GCF of 8. Factoring (dividing) a 8 out of \(8x + 16\) results in

\[
8 \left( \frac{8x}{8} + \frac{16}{8} \right)
\]

When we reduce each fraction we get \(8(x + 2)\)

The factored form of \(8x + 16\) is \(8(x + 2)\)

Check: You can check your answer by distributing the GCF back in.

Example 2. Factor the GCF out of \(15x^2 - 10x\)

\(15x^2 - 10x\) has a GCF of \(5x\). Factoring (dividing) a \(5x\) out of \(15x^2 - 10x\) results in

\[
5x \left( \frac{15x^2}{5x} - \frac{10x}{5x} \right)
\]

When we reduce each fraction we get \(5x(3x - 2)\)

The factored form of \(15x^2 - 10x\) is \(5x(3x - 2)\)

Example 3

Factor
\[
6y^2 - 10y \quad \text{(GCF is } 2y)\]

\[
2y \left( \frac{6y^2}{2y} - \frac{10y}{2y} \right) = 2y(3y - 5)
\]

Example 4

Factor
\[
12x^3 + 4x^2 \quad \text{(GCF is } 4x^2)\]

\[
4x^2 \left( \frac{12x^3}{4x^2} + \frac{4x^2}{4x^2} \right) = 4x^2(3x + 1)
\]

Example 5

Factor
\[
12y^4 + 27y^3 - 6y^2 \quad \text{(GCF is } 3y^2)\]

\[
3y^2 \left( \frac{12y^4}{3y^2} + \frac{27y^3}{3y^2} - \frac{6y^2}{3y^2} \right) = 3y^2(4y^2 + 9y - 2)
\]
## Factoring Out the GCF

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<tr>
<th>Example 6</th>
<th>Example 7</th>
<th>Example 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor: $12x^2 - 10x + 6$&lt;br&gt;(the GCF is 2)</td>
<td>Factor: $5x^2y + 20xy - 5y$&lt;br&gt;(the GCF is 5y)</td>
<td>Factor: $10x^3 - 20x^2 + 50x$&lt;br&gt;(the GCF is 10x)</td>
</tr>
<tr>
<td>$= 2(6x^2 - 5x + 3)$</td>
<td>$= 5y(x^2 + 4x - 1)$</td>
<td>$= 10x(x^2 - 2x + 5)$</td>
</tr>
</tbody>
</table>