Section 9 – 9: Solving Quadratic Equations by the use of the Square Root Property

Solving Quadratic Equations

Chapter 7 introduced the solution of second degree equations of the form $Ax^2 + Bx + C = 0$ by factoring and the use of the Zero Factor Rule. This is the simplest way to find the two solutions to a quadratic equation if the second degree equation can be factored.

The last section introduced the Quadratic Formula method to solve second degree equations of the form $Ax^2 + Bx + C = 0$ if the second degree expression cannot be factored or you do not want to solve by factoring. The Quadratic Formula can be used to solve ALL Quadratic Equations but it requires more work than the factoring method.

If $B = 0$ in the Quadratic Equation $Ax^2 + Bx + C = 0$ then the Quadratic Equation will have the form $Ax^2 + C = 0$. Equations of the form $Ax^2 + C = 0$ can be solved by the Quadratic Equation method but there is an alternate method that may prove more desirable. This method is based on the Principle of Square Roots.

The Principle of Square Roots

If $x^2 = K$ and $K$ is a positive number then

$$\sqrt{x^2} = \pm \sqrt{K}$$

and $x = \sqrt{K}$ or $x = -\sqrt{K}$

If $x^2 = K$ and $K$ is a NEGITIVE number then there are NO REAL NUMBERS for $x$ that are solutions to $x^2 = K$.

NRN

Example 1

Solve: $x^2 = 25$

$$\sqrt{x^2} = \pm \sqrt{25}$$

$x = \pm 5$

Example 2

Solve: $x^2 = -7$

$$\sqrt{x^2} = \pm \sqrt{-7}$$

$\sqrt{-7}$ is not a real number so NRN
Solving Quadratic Equations of the form $Ax^2 + C = 0$

Step 1: Solve $Ax^2 + C = 0$ for the $x^2$ quantity.

Step 2. Use the Principle of Square Roots to solve for $x$.

The Principle of Square Roots

If $x^2 = K$
and $K$ is a positive number
then
$$\sqrt{x^2} = \pm \sqrt{K}$$
and
$$x = \sqrt{K} \text{ or } x = -\sqrt{K}$$

If $x^2 = K$
and $K$ is a NEGATIVE number
then there are NO REAL NUMBERS for $x$
that are solutions to $x^2 = K$

NRN

Example 3

Solve: $x^2 - 3 = 0$

Solve for $x^2$

$x^2 = 3$

take the square root of both sides

do the equation and add a $\pm$ sign

$$\sqrt{x^2} = \pm \sqrt{3}$$

$$x = \pm \sqrt{3}$$

$x = +\sqrt{3} \text{ or } x = -\sqrt{3}$

Check: $x = \sqrt{3}$

Check: $x = -\sqrt{3}$

$$\left(\sqrt{3}\right)^2 = 3 \quad \left(-\sqrt{3}\right)^2 = 3$$

$$3 = 3 \quad 3 = 3$$

Example 4

Solve: $x^2 - 8 = 0$

Solve for $x^2$

$x^2 = 8$

take the square root of both sides

do the equation and add a $\pm$ sign

$$\sqrt{x^2} = \pm \sqrt{8}$$

$$x = \pm \sqrt{8}$$

$x = \pm \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

$x = +2\sqrt{2} \text{ or } x = -2\sqrt{2}$

Check: $x = 2\sqrt{2}$

Check: $x = -2\sqrt{2}$

$$\left(2\sqrt{2}\right)^2 - 8 = 0 \quad \left(-2\sqrt{2}\right)^2 - 8 = 0$$

$$8 - 8 = 0 \quad 8 - 8 = 0$$
Example 5
Solve: \(4x^2 - 3 = 0\)

Solve for \(x^2\)

\[x^2 = \frac{3}{4}\]

take the square root of both sides of the equation and add a \(\pm\) sign

\[\sqrt{x^2} = \pm \frac{\sqrt{3}}{\sqrt{4}}\]

\[x = \pm \frac{\sqrt{3}}{2}\]

\[x = \pm \frac{\sqrt{3}}{2}\] or \[x = -\frac{\sqrt{3}}{2}\]

Check: \(x = \frac{\sqrt{3}}{2}\)

\[4 \left(\frac{\sqrt{3}}{2}\right)^2 - 3 = 0\]

\[4 \left(\frac{3}{4}\right) - 3 = 0\]

\[3 - 3 = 0\]

Check: \(x = -\frac{\sqrt{3}}{2}\)

\[4 \left(-\frac{\sqrt{3}}{2}\right)^2 - 3 = 0\]

\[4 \left(-\frac{3}{4}\right) - 3 = 0\]

\[3 - 3 = 0\]

Example 6
Solve: \(x^2 + 16 = 0\)

Solve for \(x^2\)

\[x^2 = -16\]

take the square root of both sides of the equation

\[\sqrt{x^2} = \sqrt{-16}\]

we require the number under the square root to be a positive number

There are no Real Numbers that work so we write NRN
Example 7

Solve: \((x - 3)^2 - 5 = 0\)

Solve for \((x + 3)^2\)

\[(x - 3)^2 = 5\]

take the square root of both sides of the equation and add a \(\pm\) sign

\[\sqrt{(x - 3)^2} = \pm\sqrt{5}\]

\[x - 3 = \pm\sqrt{5}\]

solve for \(x\)

\[x = 3 + \sqrt{5} \quad \text{or} \quad x = 3 - \sqrt{5}\]

Example 8

Solve: \((x + 4)^2 - 12 = 0\)

Solve for \((x + 4)^2\)

\[(x + 4)^2 = 12\]

take the square root of both sides of the equation and add a \(\pm\) sign

\[\sqrt{(x + 4)^2} = \pm\sqrt{12}\]

\[x + 4 = \pm\sqrt{12}\]

solve for \(x\)

\[x = -4 + 2\sqrt{3} \quad \text{or} \quad x = -4 - 2\sqrt{3}\]
Example 9

Solve: \((2x - 5)^2 - 7 = 0\)

Solve for \((2x - 5)^2\)

\((2x - 5)^2 = 7\)

take the square root of both sides of the equation and add a \(\pm\) sign

\[\sqrt{(2x - 5)^2} = \pm\sqrt{7}\]

\(2x - 5 = \pm\sqrt{7}\)

solve for \(x\)

\(2x = 5 \pm \sqrt{7}\)

\(x = \frac{5 \pm \sqrt{7}}{2}\)

Example 10

Solve: \((3x + 7)^2 + 16 = 0\)

Solve for \((3x + 7)^2\)

\((3x + 7)^2 = -16\)

take the square root of both sides of the equation

\[\sqrt{(3x + 7)^2} = \sqrt{-16}\]

we require the number under the square root to be a positive number

There are no Real Numbers that work so we write \(\text{NR}\)