Section 6 – 2: Negative Exponents

A Variable with a Negative Exponent

Changing a base with a Negative Exponent into a base with a Positive Exponent.

If \( x \) is any non zero number then \( x^{-1} = \frac{1}{x} \) and \( \frac{1}{x^{-1}} = x \)

If a base has a negative exponent then it must be moved. If the base with the negative exponent is in the numerator (top) of a fraction it must be moved to the denominator (bottom) and the exponent made positive. If the base with the negative exponent is in the denominator (bottom) of a fraction it must be moved to the numerator (top) and the exponent made positive.

**Example 1**

\[
x^{-2} = \frac{1}{x^2}
\]

the base of \( x \) has a negative exponent

\( x^{-2} \) must be moved to the bottom and the exponent made positive

**Example 2**

\[
\frac{y^{-4}}{x^2} = \frac{1}{x^2 y^4}
\]

the base of \( y \) has a negative exponent

\( y^{-4} \) must be moved to the bottom and the exponent made positive

**Example 3**

\[
\frac{1}{x^{-4}} = x^4
\]

the base of \( x \) has a negative exponent

\( x^{-4} \) must be moved to the top and the exponent made positive

**Example 4**

\[
\frac{1}{x^{-5} y^3} = \frac{x^5}{y^3}
\]

the base of \( x \) has a negative exponent

\( x^{-5} \) must be moved to the top and the exponent made positive

**Example 5**

\[
\frac{y^{-3}}{x^2} = \frac{x^2}{y^3}
\]

\( y^{-3} \) must move to the bottom

\( x^{-2} \) must move to the top
A Constant with a Negative Exponent

A Constant with a negative exponent is treated just like a variable with a negative exponent.

A constant with a negative number in front **DOES NOT MOVE.**

**Example 7**

\[ 3^{-2} \]

the base of 3 has a negative exponent

\[ 3^{-2} \] must be moved to the bottom

and the exponent made positive

\[ 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \]

**Example 8**

\[ \frac{1}{2^{-3}} \]

the base of 2 has a negative exponent

\[ 2^{-3} \] must be moved to the top

and the exponent made positive

\[ \frac{1}{2^{-3}} = 2^3 = 8 \]

**Example 9**

\[ \frac{3^{-2}}{4} \]

the base of 3 has a negative exponent

\[ 3^{-2} \] must be moved to the bottom

and the exponent made positive

\[ \frac{3^{-2}}{4} = \frac{1}{4 \cdot 3^2} = \frac{1}{36} \]

**Example 10**

\[ \frac{2^{-3}}{5^{-2}} \]

\[ 2^{-3} \] must move to the bottom

\[ 5^{-2} \] must move to the top

\[ \frac{2^{-3}}{5^{-2}} = \frac{5^2}{2^3} = \frac{25}{8} \]

**Sample Problems**

**Simplify. Leave no negative exponents in your answer**

**Example 11**

\[ \frac{-3x}{2x^{-4}} \]

\[ = \frac{-3x \cdot x^4}{2} \]

\[ = -\frac{3x^5}{2} \]

**Example 12**

\[ \frac{2^{-3} y^{-2}}{y^6} \]

\[ = \frac{1}{2^3 y^6 \cdot y^2} \]

\[ = \frac{1}{8y^8} \]

**Example 13**

\[ \frac{3y^{-7}}{4^{-1} x^2} \]

\[ = \frac{3 \cdot 4}{y^7 x^2} \]

\[ = \frac{12}{y^7 x^2} \]

**Example 14**

\[ \frac{3^{-2} y^{-5}}{x^{-4}} \]

\[ = \frac{x^4}{3^2 y^5} \]

\[ = \frac{x^4}{9y^5} \]
Power Rule, Product Rule, Negative Exponents, Quotient Rule

Putting them all together

The order of operations **PEMDAS** requires that we **perform the Power Rule first.** After the power rule has been performed there are several options for the order of the remaining rules to be performed. A common order is listed below.

**Step 1:** **The Power Rule must be done first.** This is a requirement of PEMDAS

**Step 2:** Perform the **Product Rule** if there is a product of common variables.

**Step 3:** Move a base with a **Negative Exponent.**

**Step 4:** Perform the **Quotient Rule** If there is a common variable on the top and bottom of the fraction.

Some students will move a base with a negative exponent second. They will then use what ever combination of product and quotient rules are needed to finish the process. Either of the suggested orders will work but **the Power Rule must always be performed first.**

**Example 1**
Step 1. Negative Exponents
Step 2. Quotient Rule

\[
\frac{x^{-7}y^{-4}}{x^{-4}y^{-9}}
\]

move the bases with negative exponents

\[
= \frac{x^4y^9}{x^7y^4}
\]

use the quotient rule for

\[
\frac{x^4}{x^7} \text{ and } \frac{y^9}{y^4}
\]

\[
= \frac{y^5}{x^3}
\]

**Example 2**
Step 1. Negative Exponents
Step 2. Quotient Rule

\[
\frac{x^5y^{-1}}{3^{-2}x^6y^{-7}}
\]

move the bases with negative exponents

\[
= \frac{3^2x^5y^7}{x^6y}
\]

use the quotient rule for

\[
\frac{x^5}{x^6} \text{ and } \frac{y^7}{y}
\]

\[
= 9y^6x
\]
Example 3
1. Power Rule
2. Negative Exponents

\[
(2xy^{-2})^{-3}
\]

perform the Power Rule by multiplying each exponent inside by the exponent outside

\[
(2^1x^1y^{-2})^{-3}
\]

= \(2^{-3}x^{-3}y^6\)

move the bases with negative exponents

= \(\frac{y^6}{8x^3}\)

Example 4
1. Power Rule
2. Negative Exponents

\[
\frac{1}{(3x^2y^{-3})^{-2}}
\]

perform the Power Rule by multiplying each exponent inside by the exponent outside

\[
\frac{1}{(3^1x^2y^{-3})^{-2}}
\]

= \(\frac{1}{3x^{-4}y^6}\)

move the bases with negative exponents

= \(\frac{9x^4}{y^6}\)

Example 5
1. Product Rule
2. Negative Exponents

\[
(5x^{-2}y^5)(-3x^{-5}y^{-2})
\]

perform the Product Rule

\[-15x^{-2-5}y^{5-2}\]

= \(-15x^{-7}y^3\)

move the bases with negative exponents

= \(-15y^3\)

Example 6
1. Product Rule
2. Negative Exponents

\[
\frac{1}{(2x^{-3}y^{-6})(4x^5y^{-3})}
\]

perform the Product Rule

\[
\frac{1}{8x^{-3+5}y^{-6-3}}
\]

= \(\frac{1}{8x^2y^{-9}}\)

move the bases with negative exponents

= \(\frac{y^9}{8x^2}\)
Example 8
1. Product Rule
2. Negative Exponents
3. Quotient Rule and Product Rule
\[
\frac{2x^4y^{-2}}{(3x^8y^{-2})(4x^{-2}y^{-3})}
\]
\[
= \frac{2x^4y^{-2}}{12x^6y^{-5}}
\]
\[
= \frac{2x^4y^5}{12x^6y^2}
\]
\[
= \frac{y^3}{6x^2}
\]

Some students will move a base with a negative exponent second. They then use what ever combination of product and quotient rules are needed to finish the process. Either of the suggested orders will work but the **power Rule must always be performed first.**

Example 9
1. Power rule
2. Negative Exponents
3. Product Rule
\[
\frac{(x^2y^{-3})^{-2}}{(xy^{-3})^3}
\]
\[
= \frac{x^{-4}y^6}{x^3y^{-9}}
\]
\[
= \frac{y^6y^9}{x^4x^3}
\]
\[
= \frac{y^{15}}{x^7}
\]

Example 10
1. Power Rule
2. Negative Exponents
3. Quotient Rule and Product Rule
\[
\frac{(3x^{-4}y^{-1})^2}{(x^2y^{-3})^{-3}}
\]
\[
= \frac{9x^{-8}y^{-2}}{x^{-6}y^9}
\]
\[
= \frac{9x^6}{x^8y^2y^9}
\]
\[
= \frac{9}{x^2y^{11}}
\]

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