Section 4 – 5B : Graphing Linear Inequalities

Graphing a Linear Inequality based on the Linear Equation \( y = mx + b \)

A linear Inequality based on the Linear Equation \( y = mx + b \) can take one of the following 4 forms

\[
\begin{align*}
y &> mx + b \\
y &< mx + b \\
y &\geq mx + b \\
y &\leq mx + b
\end{align*}
\]

Step 1: Graph the boundary as a solid or dashed line.

A. If the inequality symbol is a \( \leq \) or \( \geq \) graph a **solid line** at \( y = mx + b \)

B. If the inequality symbol is a \( < \) or \( > \) graph a **dashed line** at \( y = mx + b \)

Step 2: Shade the area above or below the boundary.

A. If the inequality symbol reads \( y \) is less than \( y < mx + b \)
   or \( y \) is less than or equal to \( y \leq mx + b \)
   then shade the area **below the** \( y = mx + b \) **line**

B. If the inequality symbol reads \( y \) is greater than \( y > mx + b \)
   or \( y \) is greater than or equal to \( y \geq mx + b \)
   then shade the area **above the** \( y = mx + b \) **line**
Example 1
\[ y \leq \frac{3}{2}x - 1 \]
(y is less than or equal to \( \frac{3}{2}x - 1 \))

**Step 1**
Graph a solid line at \( y = \frac{3}{2}x - 1 \)

**Step 2**
Shade the area below the solid boundary line

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Check your answer:

Pick any point in the shaded area and test to see if it is a solution to \( y \leq \frac{3}{2}x - 1 \) The selected point shown is (4, 1)

Is (4, 1) a solution to \( y \leq \frac{3}{2}x - 1 \)?

Plug \( x = 4 \) and \( y = 1 \) into \( y \leq \frac{3}{2}x - 1 \)

\[ 1 \leq \frac{3}{2}(4) - 1 \]
\[ 1 \leq 5 \]
which is TRUE

The point BELOW the solid line is a solution so All points BELOW the solid line will be a solution.

Pick any point in the unshaded area and test to see if it is a solution to \( y \leq \frac{3}{2}x - 1 \) The selected point shown is (2, 4)

Is (2, 4) a solution to \( y \leq \frac{3}{2}x - 1 \)?

Plug \( x = 2 \) and \( y = 4 \) into \( y \leq \frac{3}{2}x - 1 \)

\[ 4 \leq \frac{3}{2}(2) - 1 \]
\[ 4 \leq 2 \]
which is NOT TRUE

The point ABOVE the solid line is NOT a solution so NO points ABOVE the solid line will be a solution.
Example 2

**Graph** \( y \geq \frac{1}{3}x \) (y is greater than or equal to \( \frac{1}{3}x \))

**Step 1**
Graph a solid line at \( y = \frac{1}{3}x \)

**Step 2**
Shade the area above the solid boundary line

Check your answer:

Pick any point in the shaded area and test to see if it is a solution to \( y \geq \frac{1}{3}x \) The selected point shown is (1, 2)

Is (1, 2) a solution to \( y \geq \frac{1}{3}x \)

Plug \( x = 1 \) and \( y = 2 \) into \( y \geq \frac{1}{3}x \)

\[ 2 \geq \frac{1}{3} \] (1) reduces to \( 2 \geq \frac{1}{3} \) which is TRUE

The point above the solid line is a solution so all points above the solid line will be a solution.

Pick any point in the unshaded area and test to see if it is not a solution to \( y \geq \frac{1}{3}x \) The selected point shown is (3, 0)

Is (3, 0) a solution to \( y \geq \frac{1}{3}x \)

Plug \( x = 3 \) and \( y = 0 \) into \( y \geq \frac{1}{3}x \)

\[ 0 \geq \frac{1}{3} \] (3) reduces to \( 0 \geq 1 \) which is NOT TRUE

The point below the solid line is NOT a solution so no points below the solid line will be a solution.
Example 3
\[ y \leq \frac{-5}{3}x + 2 \]
(y is less than or equal to \(\frac{-5}{3}x + 2\))

Step 1
Graph a solid line at \( y = \frac{-5}{3}x + 2 \)

Step 2
Shade the area below the solid boundary line

Example 4
\[ y \geq \frac{-1}{2}x + 1 \]
(y is greater than or equal to \(\frac{-1}{2}x + 1\))

Step 1
Graph a solid line at \( y = \frac{-1}{2}x + 1 \)

Step 2
Shade the area above the solid boundary line
Example 5

Graph \( y < 2x - 1 \)

(\( y \) is less than \( 2x - 1 \))

Step 1
Graph a dashed line at \( y = 2x - 1 \)

Step 2
Shade the area below the dashed boundary line

Example 6

Graph \( y > - \frac{1}{2} x + 3 \)

\( y \) is less than \( y > - \frac{1}{2} x + 3 \)

Step 1
Graph a dashed line at \( y = - \frac{1}{2} x + 3 \)

Step 2
Shade the area above the dashed boundary line
If the inequality is not solved for y then the first step will require that you solve for y.

In all the examples above each inequality was already solved for y. If the inequality has been solved for y then the less than symbol \((y < mx+b)\) and the less than or equal to symbol \((y \leq mx+b)\) tells you to shade **below the boundary line**.

If the inequality has been solved for y then the greater than symbol \((y > mx+b)\) and the greater than or equal to symbol \((y \geq mx+b)\) tells you to shade **above the boundary line**.

**Reminder:** If you ever multiply or divide both sides of the inequality by a negative number you must **switch the direction of the inequality symbol**.

**Example 7**

**Step 1**
Solve for y

\[-4x - 3y < -3\]

\[-4x - 3y < -3\]

\[-4x + 4x\]

\[-3y < 4x - 3\]

Divide by a negative switch the inequality

\[\frac{-3y}{-3} > \frac{4x}{-3} - \frac{3}{-3}\]

\[y > -\frac{4}{3}x + 1\]

**Step 2**
Graph a dashed line at

\[y > -\frac{4}{3}x + 1\]

**Step 3**
Shade the area **above** the dashed boundary line

**Note:** When you divided by a \(-3\) the direction of the inequality symbol had to be switched.
Example 8

Graph \(-3x + 2y < -4\)

**Step 1**
Solve for \(y\)

\[-3x + 2y < -4\]

\[-3x + 2y < -4\]

\[+3x \quad +3x\]

\[2y < 3x - 4\]

\[\frac{2y}{2} < \frac{3x - 4}{2}\]

\[y < \frac{3}{2}x - 2\]

**Step 2**
Graph a dashed line at

\[y < \frac{3}{2}x - 2\]

**Step 3**
Shade the area below the dashed boundary line

Note: **You did NOT divide by a negative** so the direction of the inequality **was not** switched.

Example 9

**Step 1**
Solve for \(y\)

\[5x - 2y \geq 6\]

\[5x - 2y \geq 6\]

\[-5x \quad -5x\]

\[-2y \geq -5x + 6\]

\[\frac{-2y}{-2} \geq \frac{-5x + 6}{-2}\]

\[y \leq \frac{5}{2}x - 3\]

**Step 2**
Graph a solid line at

\[y \leq \frac{5}{2}x - 3\]

**Step 3**
Shade the area below the below boundary line

Note: **When you divided by a \(-2\)** the **direction of the inequality** had to be switched.
Graphing a Linear Inequality based on the Linear Equation $y = \text{constant}$

A linear inequality based on the linear equation $y = \text{constant}$ can take one of the following 4 forms

- $y > \text{constant}$
- $y > \text{constant}$
- $y < \text{constant}$
- $y < \text{constant}$
- $y \leq \text{constant}$
- $y \leq \text{constant}$

**Step 1:** Graph the boundary as a solid or dashed line.

A. If the inequality symbol is a $\leq$ or $\geq$ graph a **horizontal solid line** at $y = \text{constant}$

B. If the inequality symbol is a $<$ or $>$ graph a **horizontal dashed line** at $y = \text{constant}$

**Step 2:** Shade the area above or below the boundary.

A. If the inequality symbol reads *$y$ is less a constant* ($y < \text{constant}$)  
   or *$y$ is less than or equal to a constant* ($y \leq \text{constant}$)  
   then shade the area **below the** $y = \text{constant}$ line.

B. If the inequality symbol reads *$y$ is greater than a constant* ($y > \text{constant}$)  
   or *$y$ is greater than or equal to a constant* ($y \geq \text{constant}$)  
   then shade the area **above the** $y = \text{constant}$ line.
**Example 10**

**Graph** $y \leq 4$

*(y is less than or equal to 4)*

**Step 1**

Graph a solid line at $y = 4$

**Step 2**

Shade the area **below** the solid boundary line

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**Check your answer:**

Pick any point *in the shaded area* and test to see if it is a solution to $y \leq 4$

The selected point shown is $(2, 1)$

Is $(2, 1)$ a solution to $y \leq 4$?

Plug $y = 1$ into $y \leq 4$

$1 \leq 4$ is true

so the point **BELOW** the solid line is a solution.

All points **BELOW** the solid line will be a solution.

Pick any point *NOT IN THE SHADED AREA* and test to see if it is **NOT A SOLUTION** to $y \leq 4$

The selected point shown is $(1, 5)$

Is $(1, 5)$ a solution to $y \leq 4$?

Plug $y = 5$ into $y \leq 4$

$5 \leq 4$ is **NOT TRUE**

so the point **ABOVE** the solid line is **NOT** a solution.

**NO points** ABOVE the solid line will be a solution.
Example 11

Graph $y \geq -3$

(y is greater than or equal to $-3$)

Step 1
Graph a solid line at $y = -3$

Step 2
Shade the area above the solid boundary line

Check your answer:

Pick any point in the shaded area and test to see if it is a solution to $y \geq -3$

The selected point shown is (0, 0)

Is (0, 0) a solution to $y \geq -3$?

Plug $y = 0$ into $y \geq -3$

$0 \geq -3$ is true

so the point ABOVE the solid line is a solution.

All points ABOVE the solid line will be a solution.

Pick any point NOT IN THE SHADED AREA and test to see if it is NOT A SOLUTION to $y \geq -3$

The selected point shown is (2, -5)

Is (2, -5) a solution to $y \geq -3$?

Plug $y = -5$ into $y \geq -3$

$-5 \geq -3$ is NOT TRUE

so the point BELOW the solid line is NOT a solution.

NO points BELOW the solid line will be a solution.
Example 12

graph $y > 1$

($y$ is greater than 1)

Step 1
Graph a dashed line at $y = 1$

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Step 2
Shade the area above the dashed boundary line

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Example 13

graph $y < -2$

($y$ is less than $-2$)

Step 1
Graph a dashed line at $y = -2$

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Step 2
Shade the area below the dashed boundary line

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Graphing a Linear Inequality based on the

Linear Equation $x = \text{constant}$

A linear Inequality based on the Linear Equation $x = \text{constant}$ can take one of the following 4 forms

$x > \text{constant}$  \hspace{1cm}  $x \geq \text{constant}$  \hspace{1cm}  $x < \text{constant}$  \hspace{1cm}  $x \leq \text{constant}$

**Step 1:** Graph the boundary as a solid or dashed line.

   A. If the inequality symbol is a $\leq$ or $\geq$ graph a **vertical solid line** at $x = \text{constant}$
   
   B. If the inequality symbol is a $<$ or $>$ graph a **vertical dashed line** at $x = \text{constant}$

**Step 2:** Shade the area to the left or to the right the boundary.

   A. If the inequality symbol reads $x$ is less than a constant ($x < \text{constant}$) or $x$ is less than or equal to a constant ($x \leq \text{constant}$)
      then shade the area **to the left of the** $x = \text{constant}$ line.
   
   B. If the inequality symbol reads $x$ is greater than a constant ($x > \text{constant}$) or $x$ is greater than or equal to a constant ($x \geq \text{constant}$)
      then shade the area **to the right of the** $x = \text{constant}$ line.
Example 14

\[ x \geq 2 \]  
(x is greater than or equal to 2)

**Step 1**
Graph a solid line at \( x = 2 \)

**Step 2**
Shade the area to the right of the solid boundary line

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**Check your answer:**
Pick any point in the shaded area and test to see if it is a solution to \( x \geq 2 \)  the selected point shown is (3, 1)

Is (3, 1) a solution to \( x \geq 2 \)

Plug \( x = 3 \) into \( x \geq 2 \)

\[ 3 \geq 2 \] so the point TO THE RIGHT of the solid line is a solution.

All points TO THE RIGHT of the solid line will be a solution.

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**Check your answer:**
Pick any point NOT IN THE SHADED AREA and test to see if it is NOT A SOLUTION to \( x \geq 2 \)

The selected point shown is (1, 2)

Is (1, 2) a solution to \( x \geq 2 \)

Plug \( x = 1 \) into \( x \geq 2 \)

\[ 1 \geq 2 \] is NOT TRUE so the point TO THE LEFT of the solid line is NOT a solution.

NO points TO THE LEFT of the solid line will be a solution.
**Example 15**

graph \( x \leq -1 \)

(x is less than or equal to \(-1\))

**Step 1**

Graph a solid line at \( x = -1 \)

**Step 2**

Shade the area to the left of the solid boundary line

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Check your answer:

Pick any point in the shaded area and test to see if it is a solution to \( x \leq -1 \) the selected point shown is \((-2, 3)\)

Is \((-2, 3)\) a solution to \( x \leq -1 \)

Plug \( x = -2 \) into \( x \leq -1 \)

\(-2 \leq -1 \) so the point TO THE LEFT of the solid line is a solution.

All points TO THE LEFT of the solid line will be a solution.

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Check your answer:

Pick any point **NOT IN THE SHADDED AREA** and test to see if it is **NOT A SOLUTION** to \( x \leq -1 \)

The selected point shown is \((0, 0)\)

Is \((0, 0)\) a solution to \( x \leq -1 \)

Plug \( x = 0 \) into \( x \leq -1 \)

\(0 \leq -1 \) is NOT TRUE so the point TO THE RIGHT of the solid line is **NOT** a solution.

NO points TO THE RIGHT of the solid line will be a solution.
Example 16
graph x > –3
(x is greater than –3)

Step 1
Graph a dashed line at x = –3

Step 2
Shade the area to the right of the dashed boundary line

Example 17
graph x < 4
(x is less than 4)

Step 1
Graph a dashed line at x = 4

Step 2
Shade the area to the left of the dashed boundary line