Section 1 – 7:  The Distributive Property for a Constant

The product of a **constant times a polynomial** is written as follows.

<table>
<thead>
<tr>
<th>a constant times</th>
<th>a constant times</th>
<th>a constant times</th>
</tr>
</thead>
<tbody>
<tr>
<td>a binominal</td>
<td>a trinominal</td>
<td>a polynomial</td>
</tr>
<tr>
<td>$5(3x + 1)$</td>
<td>$3(3x^2 + 2x - 6)$</td>
<td>$3(3x^3 + 2x^3 - 6x + 4)$</td>
</tr>
</tbody>
</table>

The **distributive property** is used to find what the product of a constant times a polynomial is.

**The Distributive Property**

$$a(bx^2 + cx + d) = (a \cdot b)x^2 + (a \cdot c)x + a \cdot d$$

The Distributive Property describes the distributive process as taking the **constant outside** the parenthesis and **multiplying** it into each term **inside** the parenthesis one product at a time.

**Example of the Distributive Property for** $2(x + 3)$

Distribute (multiply) the 2 into the **first term** of the polynomial

$2(x + 3) = 2x$

Distribute (multiply) the 2 into the **second term** of the polynomial

$2(x + 3) = 6$

Both steps done at once look like this

$2(x + 3) = 2x + 6$

If there are more **then two terms** inside the parenthesis then you multiply the constant **outside** the parenthesis into each term **inside** the parenthesis.

**Example 1**

A constant times a Binomial

$-4(2x - 5)$ means

$-4(2x) - 4(-5)$

$= -8x + 20$

**Example 2**

A constant times a Binomial

$\frac{3}{2}(-6x + 10)$ means

$\frac{3}{2}(-6x) + \left(\frac{3}{2}\right)(+10)$

$= -18 \cdot \frac{x}{2} + \frac{30}{2}$

$= -9x + 15$
Distributing a Fraction

A fraction is distributed just like an integer. Some students multiply the fraction times each integer inside the parenthesis in their head. It may help to show the multiplication of the fraction times each integer as a fraction and then reduce the fractions as a second step. This process is shown below. The integers inside the parenthesis can be written as a fraction with a 1 in the denominator. This may help some students to multiply the fractions.

Example 7

Distribute: \( \frac{3}{2}(-6x^2 + 4x) \)

\[
\begin{align*}
\text{multiply } \frac{3}{2} \text{ times } -6x^2 & \\
\text{multiply } \frac{3}{2} \text{ times } +4x & \\
\frac{3}{2}(-6x^2) \text{ and } \frac{3}{2}(+4x) & \\
= -18 \frac{x^2}{2} + 12 \frac{2}{x} & \\
= -9x^2 + 6x &
\end{align*}
\]

Example 8

Distribute: \( \frac{-4}{3}(12y-9) \)

\[
\begin{align*}
\text{multiply } \frac{-4}{3} \text{ times } 12y & \\
\text{multiply } \frac{-4}{3} \text{ times } -9 & \\
\frac{-4}{3}(12y) \text{ and } \frac{-4}{3}(-9) & \\
= -48 \frac{12y}{3} + \frac{36}{3} & \\
= -16y + 12 &
\end{align*}
\]
Distribute – Combine Like Terms

The Order of Operations requires that multiplication be done before addition or subtraction. Since the distributive process is a multiplication step it must be done before the addition or subtraction of like terms. After the distributive process is complete, the remaining polynomial may be further simplified by combining any Like Terms that occur.

First Distribute the number outside the parentheses and then combine Like Terms.

**Example 7**

\[ 5(2x - 3) + 6x \]

(distribute the 5)

\[ = 10x - 15 + 6x \]

(combine like terms)

\[ = 16x - 15 \]

**Example 8**

\[ 2 - (-6x + 7) \]

(distribute the \(-1\))

\[ = 2 + 6x - 7 \]

(combine like terms)

\[ = 6x - 5 \]

**Example 9**

\[ 9y^2 - 3(-2y^2 + 4y) \]

(distribute the \(-3\))

\[ = 9y^2 + 6y^2 - 12y \]

(combine like terms)

\[ = 15y^2 - 12y \]

**Example 10**

\[ -2(4y^2 - 3y) - 2y \]

(distribute the \(-2\))

\[ = -8y^2 + 6y - 2y \]

(combine the like terms)

\[ = 8y^2 + 4y \]

**Example 11**

\[ 2(x^2 + 3x) - 3(x^2 - 4x) \]

(distribute the 2 and the \(-3\))

\[ = 2x^2 + 6x - 3x^2 + 12x \]

(combine like terms)

\[ = -x^2 + 18x \]

**Example 12**

\[ -3(2x^2 + 1) - (2x + 5) \]

(distribute the \(-3\) and the \(-1\) )

\[ = 6x^2 - 3x - 2x - 5 \]

(combine the like terms)

\[ = 6x^2 - 5x - 5 \]