**Math 120 Intermediate Algebra**  
Sec 8.1: Quadratic Equations & Sec 8.2: The Quadratic Formula

### Methods to Solve a Quadratic Equation

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Factor Property</td>
<td>Use if you see an x term</td>
</tr>
<tr>
<td>OR</td>
<td>Square Root Property</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>Use as a LAST RESORT</td>
</tr>
<tr>
<td>Completing the Square</td>
<td>Use ONLY WHEN ASKED</td>
</tr>
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</table>

### Principle of Square Roots/Square Root Property

Let X be an algebraic expression and k be any real number.

If \(X^2 = k\) then \(X = \sqrt{k}\) or \(X = -\sqrt{k}\)

### The Quadratic Formula

The solutions of \(ax^2 + bx + c = 0, a \neq 0\), are given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

How to identify which method should be used:

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**Ex 1** Solve each quadratic equation following the guidelines above.

a) \(c^2 - 8 = 0\)  

b) \(3x^2 - 7x = -2\)  

c) \(2t(t + 2) = 1\)

d) \((4x + 5)^2 = 12\)
<table>
<thead>
<tr>
<th>Ex 2</th>
<th>Solve using the method indicated.</th>
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<tbody>
<tr>
<td>QF</td>
<td>$x^2 + 6x = 16$</td>
</tr>
<tr>
<td>ZFP</td>
<td>$x^2 + 6x = 16$</td>
</tr>
<tr>
<td>QF</td>
<td>$3x^2 + 8x + 4 = 0$</td>
</tr>
<tr>
<td>ZFP</td>
<td>$3x^2 + 8x + 4 = 0$</td>
</tr>
<tr>
<td>QF</td>
<td>$9x^2 - 133x - 30 = 0$</td>
</tr>
<tr>
<td>ZFP</td>
<td>$9x^2 - 133x - 30 = 0$</td>
</tr>
<tr>
<td>QF/ZFP</td>
<td>$(\frac{3}{2}x - 7)^2 = 16$</td>
</tr>
<tr>
<td>Square Root Property</td>
<td>$(\frac{3}{2}x - 7)^2 = 16$</td>
</tr>
</tbody>
</table>
Ex 3 Solve.

a) \( t^2 = 144 \)
b) \( 7x^2 - 5 = 0 \)
c) \( 7x^2 - 5x = 0 \)

d) \( \frac{9}{x} - 2 = \frac{5}{x^2} \)
e) \( (x + 1)^2 = 100 \)
f) \( x^2 + 11 = 16 \)

g) \( (x - 1)^2 = -18 \)
h) \( x(x - 1) = 2x - 7 \)
i) \( \left(t + \frac{3}{2}\right)^2 = \frac{7}{2} \)
j) Do
\[ x^3 + 8 = 0 \]
Consider \( x^3 = -8 \)

k) Do
\[ 25y^2 + 16 = 0 \]

l) Do
\[ 2(29x + 1)^2 = 128 \]

m) Do
\[ (29x + 4)(29x - 5) = 0 \]

n) Do
\[ (x + 1)(x + 2) = 3(x + 1) \]

o) Start
\[ (x + 1)(x + 2) = 4 \]

Ex 4  Let \( f(t) = (t + 6)^2 \). Find \( t \) such that \( f(t) = 15 \).

Ex 5  Fill in the blanks to complete the square.

a) \( x^2 + 12x + \underline{\hspace{2cm}} = (x + \underline{\hspace{1cm}})^2 \)

b) \( x^2 - 2x + \underline{\hspace{2cm}} = (x - \underline{\hspace{1cm}})^2 \)

c) \( t^2 - 9t + \underline{\hspace{2cm}} = (t - \underline{\hspace{1cm}})^2 \)

d) \( x^2 + \frac{5}{3}x + \underline{\hspace{2cm}} = (x + \underline{\hspace{1cm}})^2 \)
Completing The Square

Note: 
\[ (x + b)^2 = x^2 + 2bx + b^2 \]

The middle coefficient is 2b. Taking half of 2b and then squaring yields \( b^2 \).

I. Given a quadratic equation \( ax^2 + bx + c = 0 \), divide (if necessary) by \( a \) (the leading coefficient) to obtain a quadratic whose leading coefficient is 1. 
\[ \text{ex} \quad 2x^2 + 10x - 15 = 0 \Rightarrow x^2 + 10x - \frac{15}{2} = 0 \]

II. Bring the constant to the other side. 
\[ \text{ex} \quad x^2 + 10x = \frac{15}{2} \]

III. Divide the middle coefficient by 2 OR multiply it by \( \frac{1}{2} \). 
\[ \text{ex} \quad \frac{10}{2} = 5 \]

IV. Square the result from previous step. 
\[ \text{ex} \quad 5^2 = 25 \]

V. Add the result, from previous step, to BOTH sides of the equation. 
\[ \text{ex} \quad x^2 + 10x + 25 = \frac{15}{2} + 25 \]

VI. Factor the polynomial as a perfect square and simplify the right side. 
\[ \text{ex} \quad (x + 5)^2 = \frac{65}{2} \]

VII. Apply the Square Root Property and solve. 
\[ \text{ex} \quad (x + 5)^2 = \frac{65}{2} \Rightarrow \sqrt{(x + 5)^2} = \sqrt{\frac{65}{2}} \Rightarrow x + 5 = \pm \frac{\sqrt{65}}{\sqrt{2}} \]
\[ \Rightarrow x = -5 \pm \frac{65}{\sqrt{2}} \]

Ex 6    Solve by completing the square.

a) \( x^2 + 8x = 9 \)  

Ex 7    Solve by using the Quadratic Formula and by Completing the Square.

2\( x^2 + 12x = -1 \)  

b) \( 2x^2 - 5x - 3 = 0 \)
Complete the square to find the $x$-intercepts of the function. $f(x) = x^2 - 8x - 10$

Given $f(x)$ and $g(x)$, find all $x$ for which $f(x) = g(x)$.

**Ex 9**

<table>
<thead>
<tr>
<th>Discriminant: $D = b^2 - 4ac$</th>
<th>Number of Solution(s)</th>
<th>Type of Solution(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = 0$</td>
<td>1</td>
<td>A rational number</td>
</tr>
<tr>
<td>$D &gt; 0$</td>
<td></td>
<td>Real solutions</td>
</tr>
<tr>
<td>- $D$ is a perfect square</td>
<td>2</td>
<td>Solutions are rational</td>
</tr>
<tr>
<td>- $D$ is not a perfect square</td>
<td></td>
<td>Solutions are irrational conjugates</td>
</tr>
<tr>
<td>$D &lt; 0$</td>
<td>2</td>
<td>Complex imaginary solutions</td>
</tr>
</tbody>
</table>

Possible outcomes:

**Ex 8**

$f(x) = x + 5$

$g(x) = \frac{3}{x - 5}$
Ex 10  Determine the number and type of solutions for each equation.

a) \( x^2 - 5x + 3 = 0 \)

b) \( 4x^2 - 12x + 9 = 0 \)

c) What is b? Rest is PP

\( x^2 - 7 = 0 \)

\( 4m^2 + 7m = 0 \)

Ex 11  Write a quadratic function and equation with integer coefficients in standard form having the given numbers as roots.

a) \(-2, 8\)

b) \(-\frac{3}{4}, \text{ only solution}\)

c) \(\pm \sqrt{7}\)

d) \(\pm 3i\)

e) \(2 - \sqrt{10}, 2 + \sqrt{10}\)

f) Practice Prob  

\(5 \pm 2i\)

\(x^2 - 10x + 29 = 0\)

\(0, \frac{2}{5}\)

\(5x^2 - 2x = 0\)
Sec 8.4: Applications Involving Quadratic Equations

Ex 12  (#6)  A turbo-jet flies 50 mph faster than a super-prop plane. If a turbo-jet goes 2000 mi in 3 hr less time than it takes the super-prop to go 2800 mi, find the speed of each plane.  Long. Write small.
Ex 13 (#12) Two pipes are connected to the same tank. Working together, they can fill the tank in 4 hr. The larger pipe, working alone, can fill the tank in 6 hr less than the smaller one. How long would the smaller one take, working alone, to fill the tank?

Ex 14 (#18) Solve for $k$. ($k$ is the number of diagonals of a polygon with $k$ sides)

\[
N = \frac{k^2 - 3k}{2}
\]

Ex 15 Prac Prob (#34) Falling Distance Use $4.9t^2 + v_0t = s$

(t = time in seconds, $v_0$ = initial velocity, $s$ = height of object)

a) A ring is dropped from a helicopter at an altitude of 75 m. Approximately how long does it take the ring to reach the ground? 3.9 sec

b) A coin is tossed downwards with an initial velocity of 30m/sec from an altitude of 75 m. Approximately how long does it take the coin to reach the ground? 1.9 sec

c) Approximately how far will an object fall in 2 sec, if thrown downward at an initial velocity of 20 m/sec from a helicopter? 59.6 m
Sec 8.5: Equations Reducible to Quadratic

Solve and check answers for domain issues.

16) \( p \) \( x^4 - 17x^2 + 16 = 0 \)
Let \( u = \)
Then \( u^2 = \)

17) \( r - 2\sqrt{r} - 6 = 0 \)
Let \( u = \)
Then \( u^2 = \)

\[ x = \pm 1, \pm 4 \]

18) \( (x^2 - 2)^2 - 12(x^2 - 2) + 20 = 0 \)
Let \( u = \)
Then \( u^2 = \)

19) \( 2x^{-2} - x^{-1} - 1 = 0 \)
Let \( u = \)
Then \( u^2 = \)

\[ x = -2, 1 \]
20) \[ w^{2/3} - 2w^{1/3} - 8 = 0 \]

Let \( u = \)

Then \( u^2 = \)

21) \[ 9 \left( \frac{x + 2}{x + 3} \right)^2 - 6 \left( \frac{x + 2}{x + 3} \right) + 1 = 0 \]

Let \( u = \)

Then \( u^2 = \)

Ex 22  Find all intercepts of each function.

Practice Prob \[ f(x) = x^{1/2} - x^{1/4} - 6 \]

\[ g(x) = (3 + \sqrt{x})^2 + 3(3 + \sqrt{x}) - 10 \]

\( x = -\frac{3}{2} \)

\( x\)-intercept: \((81, 0)\); \( y\)-intercept: \((0, -6)\)
Sec 8.6: Quadratic Functions and Their Graphs

Vertex Form: \( f(x) = a(x - h)^2 + k \)  \( \text{Vertex: } (h, k) \)

The graph of \( f(x) = a(x - h)^2 + k \) has the same shape as the graph of \( y = ax^2 \).

- If \( h > 0 \), the graph of \( y = ax^2 \) is shifted \( h \) units to the right. \( \text{ex } f(x) = 2(x - 3)^2 \)
- If \( h < 0 \), the graph of \( y = ax^2 \) is shifted \( |h| \) units to the left. \( \text{ex } f(x) = 2(x - (-3))^2 = 2(x + 3)^2 \)
- If \( k > 0 \), the graph of \( y = ax^2 \) is shifted \( k \) units up. \( \text{ex } f(x) = 2x^2 + 3 \)
- If \( k < 0 \), the graph of \( y = ax^2 \) is shifted \( |k| \) units down. \( \text{ex } f(x) = 2x^2 + (-3) = 2x^2 - 3 \)
- The vertex is \((h, k)\) and the axis of symmetry is \( x = h \).
- If \( a > 0 \), the parabola opens upward and the ________ function value is \( k \).
- If \( a < 0 \), the parabola opens downward and the ________ function value is \( k \).

Ex 23 Graph the function, label the vertex, draw the axis of symmetry, and find the maximum/minimum value (extremum).

a) \( f(x) = -\frac{1}{4}x^2 \)

b) \( g(x) = 3(x - 5)^2 \)

c) \( h(x) = -(x - 1)^2 + 2 \)

d) \( r(x) = \frac{3}{2}(x + 2)^2 - 4 \)
Ex 24 Without graphing, find the vertex, the axis of symmetry, and the extremum.

a) \( f(x) = 2(x - 1)^2 - 10 \)  

b) \( f(x) = -2\pi(x + 0.01)^2 + \sqrt{15} \)

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Ex 25 Write a function having a graph with the same shape as the graph of \( f(x) = \frac{3}{5}x^2 \), but with the given point as the vertex.

a) \((9, -6)\)  
b) Practice Prob \((-1, \frac{2}{5})\)

\[ g(x) = \frac{3}{5}(x + 1)^2 + \frac{2}{5} \]

Ex 26 Write the equation of the parabola that has the shape of \( f(x) = 2x^2 \) or \( g(x) = -2x^2 \) and has a maximum/minimum value at the specified point.

Maximum; \((-4, 0)\)

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**Sec 8.7: More About Graphing Quadratic Functions**

Finding the vertex of a parabola:
The Vertex of a Parabola

The vertex of the parabola given by \( f(x) = ax^2 + bx + c, a \neq 0 \) is \( \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \) or \( \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) \).

- The \( x \)-coordinate of the vertex is \(-\frac{b}{2a}\).
- The axis of symmetry is \( x = -\frac{b}{2a} \).
- The second coordinate of the vertex is most commonly found by computing \( f\left(-\frac{b}{2a}\right) \).

**Ex 27** Complete the square to write each function in vertex form. Next, find the vertex.

a) \( f(x) = x^2 + 5x + 3 \)  

Do again using vertex formula

b) \( f(x) = -2x^2 - 5x + 10 \)

**Ex 28** For each quadratic function, (a) find the vertex and axis of symmetry, (b) find the maximum/minimum function value, and (c) graph the function.

a) \( f(x) = x^2 - 2x - 3 \)  
b) \( f(x) = -3x^2 - 7x + 2 \)

Write in vertex form using CTS method and vertex formula.
c) (#42) Find the vertex by CTS (similar to exam, \( a = \frac{1}{3} \))

\[ h(x) = \frac{1}{2}x^2 - 3x + 2 \]

**Ex 29** Let \( f(x) = x^2 - 10x + 21 \). Find the vertex and all intercepts; use this information to graph the function. Draw and label the axis of symmetry.
Ex 30 Suppose that a flare is launched upward with an initial velocity of 128 ft/sec from a height of 144 ft. Its height in feet, $h(t)$ after $t$ seconds is given by: $h(t) = -16t^2 + 128t + 144$.

a) Find the height of the object after 2 seconds.

b) How many seconds will it take the object to hit the ground?

c) When will it reach its maximum height? How high will the flare go?

Extra Credit: Vertex of a Parabola Handout

Parabolas ‘Changing Shape’ Handout pg 59 (~3 probs) 15 min
The Flare Problem II pgs 65-66 (Do #2, complete remaining for practice) 10 min

Sec 8.9: Polynomial and Rational Inequalities
Ex 31 Solve each inequality using the graph provided. Express answers in interval notation.

a) (#8) $p(x) < 0$
b) (#10) 
\[ x^4 + x^3 \geq 6x^2 \]

Ex 32  Solve.

a) (#14) 
\[(x + 8)(x + 10) > 0\]

b) (#18') 
\[x^2 + x - 2 \leq 0\]

c) (#20') 
\[x^2 + 6x + 9 \leq 0\]

d) (#22) PP (Start) 
\[x^2 + 6x \geq 2\]
e) (#24)
\[5x(x + 1)(x - 1) > 0\]

f) (#40)
\[\frac{(x + 4)(x - 1)}{x + 3} \geq 0\]

**Ex 33**  Prac Prob  (#30)
(\(-\infty, -3\) ∪ (-1, 2))

For \(g(x) = (x + 3)(x - 2)(x + 1)\), find all values of \(x\) for which \(g(x) < 0\).

**Ex 34**  Solve.
\[x^2 \leq 9\]
\[x^2 \geq 9\]

\[x^2 \leq -9\]
\[x^2 \geq -9\]
Sec 8.8: Problem Solving and Quadratic Functions

Ex 35 (#36) Find a quadratic function that fits the set of data points. \((1, 4), (-1, 6), (-2, 16)\).

Optimization Applications pg 67 (Do #1, remaining is hw)

Ex 36 (#22') What is the maximum product of two numbers that add to 17? What numbers yield this product? If time permits/Prac Prob 289/4: 17/2 and 17/2

Max Product:

Numbers that Yield Max Product:

Start Func Worksheet VII
Ex 37 (#13) A stone mason has enough stones to enclose a rectangular patio with 60 ft of perimeter, assuming that the attached house forms one side of the rectangle. What is the maximum area that the mason can enclose? What should the dimensions of the patio be in order to yield this area?

If time permits/Prac Prob 15 ft by 30 ft

Ex 38 (#15) Economite Plastics plans to produce a one-compartment vertical file by bending the long side of an 8-in. by 14-in. sheet of plastic along two lines to form a U shape. How tall should the file be in order to maximize the volume that the file can hold?

If time permits/Prac Prob 3.5 in