Chapter 9 Notes

Inference Tests about the Difference between Two Population Proportions

Assumptions:

1. Both samples are independently obtained from their respective populations using simple random sampling.

2. $n_1 \bullet \hat{p}_1 \geq 5$ **and** $n_1 \bullet \hat{p}_2 \geq 5$ **OR** both populations are normal.

A Hypothesis Test to Compare Claims about the Difference in 2 Population Proportions

The Test Statistic for **Two Population Proportions with** H_0: $p_1 = p_2$

$\hat{p}_1 = \dfrac{x_1}{n_1}$ and $\hat{p}_2 = \dfrac{x_2}{n_2}$ rounded off to 2 decimal places

$\overline{p} = \dfrac{(x_1 + x_2)}{(n_1 + n_2)}$ rounded off to 2 decimal places

Test Statistic: $z = \dfrac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\left(\dfrac{\overline{p} \bullet \overline{q}}{n_1}\right) + \left(\dfrac{\overline{p} \bullet \overline{q}}{n_2}\right)}}$ rounded off to 2 decimal places

Creating a Confidence Interval to Estimate the value of the Difference in 2 Population Proportions $(p_1 - p_2)$

if $\hat{p}_1 > \hat{p}_2$ **then**

$$(\hat{p}_1 - \hat{p}_2) - E \ < \ p_1 - p_2 \ < \ (\hat{p}_1 - \hat{p}_2) + E$$

or

if $\hat{p}_2 > \hat{p}_1$ **then**

$$(\hat{p}_2 - \hat{p}_1) - E \ < \ (p_2 - p_1) \ < \ (\hat{p}_2 - \hat{p}_1) + E$$

Where $E = z_{\alpha/2} \bullet \sqrt{\left(\dfrac{\hat{p}_1 \bullet \hat{q}_1}{n_1}\right) + \left(\dfrac{\hat{p}_2 \bullet \hat{q}_2}{n_2}\right)}$ rounded off to 2 decimal places

Inference Tests about the Difference between Two Population Means

Assumptions:

1. Both samples are independently obtained from their respective populations using simple random sampling.

2. $n_1 > 30$ **and** $n_2 > 30$ **OR** both populations are normal.

A Hypothesis Test to Compare Claims
about the Difference in 2 Population Means
using 2 Independent Samples

The Test Statistic for **Two Population Means with** H_0: $\mu_1 = \mu_2$

Test Statistic: $$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

with a significance level of α
and the Degree of Freedom is the smaller of the two values $(n_1 - 1)$ or $(n_2 - 1)$

Creating a Confidence Interval to estimate the value of
the Difference in 2 Population Means $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) - E \;<\; (\mu_1 - \mu_2) \;<\; (\bar{x}_1 - \bar{x}_2) + E \quad \textbf{if} \quad \bar{x}_1 > \bar{x}_2$$

or

$$(\bar{x}_2 - \bar{x}_1) - E \;<\; (\mu_2 - \mu_1) \;<\; (\bar{x}_2 - \bar{x}_1) + E \quad \textbf{if} \quad \bar{x}_2 > \bar{x}_1$$

Where $$E = t_{\alpha/2} \bullet \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$$

with a significance level of α
and the Degree of Freedom is the smaller of the two values $(n_1 - 1)$ or $(n_2 - 1)$

Inference Tests about the Difference in 2 Population Standard Deviations

1. Both samples are independently obtained from their respective populations using simple random sampling.

2. Both populations are normal.

Hypothesis Testing to Compare the Difference in 2 Population Standard Deviations with H_0: $\sigma_1 = \sigma_2$

chose the population with

the largest sample standard deviation

to be **Population One**

with a sample standard deviation of s_1 and degrees of freedom $\text{DF} = n_1 - 1$

chose the population with

the smallest sample standard deviation to be

Population Two

with a sample standard deviation of s_2 and degrees of freedom $\text{DF} = n_2 - 1$

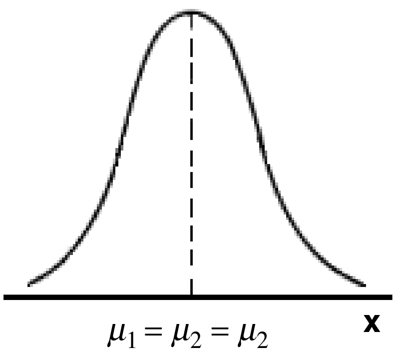
Test Statistic: $F = \dfrac{(s_1)^2}{(s_2)^2}$ where $s_1 > s_2$

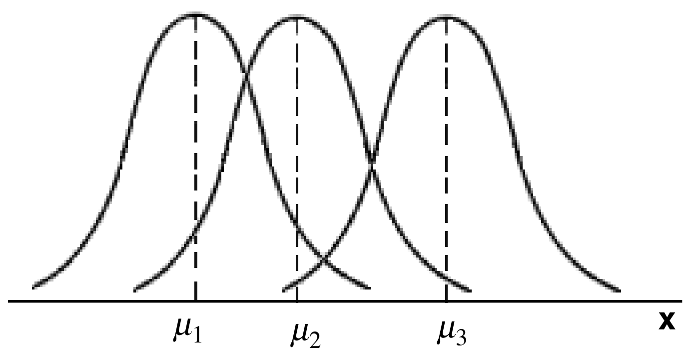
with The Numerators DF = $n_1 - 1$ and the Denominators DF = $n_2 - 1$

ne Way ANOVA: The One Way Analysis of Variance

Testing a Claim about 3 or more Population Means

H_0: $\mu_1 = \mu_2 = \mu_3$
versus
H_1: **At least one of the population mean is different from the others.**





The techniques of One Way ANOVA can be used to test if

1. **k populations** are defined with **only one factor** is used to differentiate the populations. All other factors concerning the population must be the same (or very similar) except for the one factor is used to differentiate the populations.

2. A random sample is taken form each of the k populations. The sample size for each sample should be the same but this is not required. Each sample must be independent of the others. The subjects in one sample cannot be related to the subjects in the others sample in any way. This says that **the samples must be independent.**

3. The populations must be **normally distributed.**

4. The populations must have the **same variance or standard deviation** $\sigma_1 = \sigma_2 = \sigma_3$ .... A rule of thumb is that One Way ANOVA procedures can be used if the largest sample standard deviation is no more than two times as large as the smallest sample standard deviation.

The requirements for One Way ANOVA are robust. This means that you can be close to meeting the requirements and the procedure will still provide a good test of the hypothesis.
If the following requirements are met then the techniques of One Way ANOVA can be used to test if

H_0: $\mu_1 = \mu_2 = \mu_3$
versus
H_1: At least one of the population mean is different form the others.

given k samples of size n and a desired level of significance.

Conclusion:

Reject H_0 if $\text{P Value} \le \alpha$ **Do Not Reject** H_0 if $\text{P Value} > \alpha$

Inference Tests about the Difference between Matched Pairs of Dependent Samples

Assumptions:

1. The population and sample data consists of matched pairs.
2. The samples are obtained using simple random sampling.
3. Each difference between the matched pairs was calculated the same way.
   (ie. Before – After or After – Before)
4. $n > 30$ **OR** the populations of differences is normal.

A Hypothesis Test to Compare Claims about the Difference in 2 Population Means using 2 Independent Samples
with $\mathrm{H}_0\text{:}\ \mu_d = 0$

Test Statistic: $$t = \frac{\left(\bar{d}\right)}{\left(\frac{s_d}{\sqrt{n}}\right)}$$

with $\mathrm{DF} = n - 1$

Creating a Confidence Interval to Estimate the value of the Mean of the Differences between Matched Pairs Dependent Samples

$$\bar{d} - E < \mu_d < \bar{d} + E$$

Where $E = t_{\alpha/2} \bullet \frac{s_d}{\sqrt{n}}$

with $\mathrm{DF} = n - 1$

