Section 9 – 4B Lecture

Creating a Confidence Interval to Estimate the value of the Mean of the Differences between Matched Pairs Dependent Samples

Requirements

1. A random sample of each population is taken. The sample mean for the sample of one population is s_1 and the sample mean for the sample of a second population is s_2
2. The two samples are **independent** of each other.
3. **Both populations are normal or both sample sizes are greater than 30.**

Requirements

1. The sample data consists consists of matched pairs.
2. The samples are simple random samples.
3. **The number of matched pairs is greater than 30 or the population of the differences is normal.**

Notation for the Samples of Two Population Means of Matched Pairs

d = the **individual differences** between each matched pair.

n = the number of **matched pairs in the sample**

μ_d = the average (mean) of all the differences d in the **population of matched pairs.**

$\overline{d}$ = the mean(average) of all the differences in the **sample of matched pairs**.

s_d = the standard deviation of all the differences in the **sample of matched pairs**.

Creating a Confidence Interval to Estimate the value of the Mean of the Differences between Matched Pairs Dependent Samples

$$\overline{d} - E < \mu_d < \overline{d} + E$$

Where $E = t_{\alpha/2} \bullet \frac{s_d}{\sqrt{n}}$

with $\mathrm{DF} = n - 1$

Creating a Confidence Interval to Estimate the value of the Mean of the Differences between Matched Pairs

Example 1

Folsom Lake College PE students are required to run a mile as part of their grade in their fitness class. A random group of 7 students ran the mile at the start of the semester and their times in the mile run were recorded. They were then timed in the mile run at the end of the semester and their final score was recorded their times in the mile run were recorded. Using the table below construct a 99% confidence interval for the population mean of the differences between before and after. Assume the population of differences is normal.

	A	B	C	D	E	F	G
Start of Sem	6.6	6.5	9.0	10.3	8.1	6.0	7.2
End of Sem.	6.2	7.1	9.3	10.4	7.9	6.5	7.2
Difference = d End – Start	–.4	.6	.3	.1	–.2	.5	0

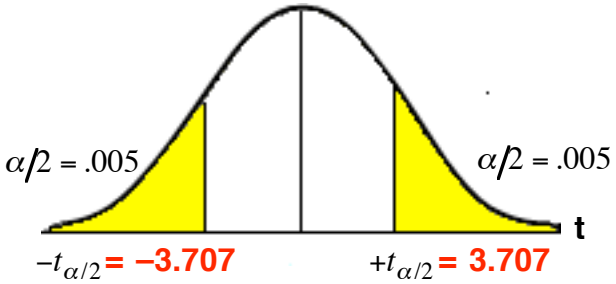
$\bar{d} = .129 \quad s_d = .364 \quad \text{DF} = 6$

$\alpha = .01 \quad \text{so} \quad \alpha/2 = .005$

Find the critical value $+t_{\alpha/2}$

$\alpha = .01$ so $\alpha/2 = .005$

DF = 6



Find the Maximum Error

$$E = t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$$

$$E = 3.707 \cdot \frac{.364}{\sqrt{7}}$$

$$E = .51$$

Confidence interval

$$\bar{d} - E < u_d < \bar{d} + E$$

$$.129 - .51 < u_d < .129 + .51$$

$$-.381 < u_d < .639$$

Conclusion based on the problem: **The confidence interval does contain zero.**

I am 99% confident that here is **no difference** in the **before and after times in the mile run.**

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
6	1.440	1.943	2.447	3.143	3.707

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Example 2

A random group of 11 students in the FLC reading class were tested at the start of the semester and the number of words they could read in one minute were recorded. They were then retested at the end of the semester and the number of words they could read in one minute were recorded. Using the table below construct a 98% confidence interval for the population mean of the differences between before and after. Assume the population of differences is normal.

	A	B	C	D	E	F	G	H	I	J	K
Before	23	18	13	42	31	19	26	25	28	24	25
After	28	26	19	56	32	28	40	37	39	40	37
Difference = d After – Before	5	.8	.6	14	1	.9	14	12	11	16	12

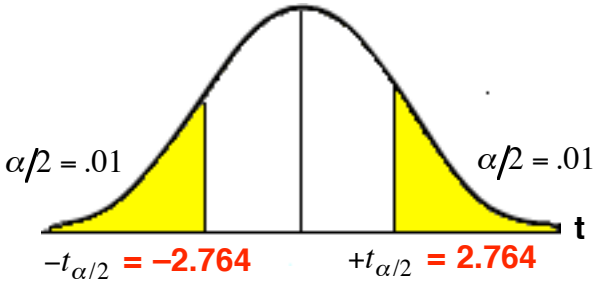
$\bar{d} = 9.83 \quad s_d = 4.51 \quad \text{DF} = 10$

$\alpha = .02 \quad \text{so} \quad \alpha/2 = .01$

Find the critical value $+t_{\alpha/2}$

$\alpha = .02$ so $\alpha/2 = .01$

DF = 10



Find the Maximum Error

$$E = t_{\alpha/2} \bullet \frac{s_d}{\sqrt{n}}$$

$$E = 2.764 \bullet \frac{4.581}{\sqrt{11}}$$

$$E = 3.82$$

Confidence interval

$$\bar{d} - E < u_d < \bar{d} + E$$

$$9.83 - 3.82 < u_d < 9.83 + 3.82$$

$$6.01 < u_d < 13.65$$

Conclusion based on the problem: **The confidence interval does NOT contain zero.**

I am 98% confident that there is a difference in the **before and after reading rates. The increase is between 6 and 13 words a minutes.**

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
10	1.372	1.812	2.228	2.764	3.169

