Testing a Claim about the Difference in 2 Population Means
Independent Samples

Test H_0: $\mu_1 = \mu_2$

(there is no difference in Population Means $\mu_1 - \mu_2 = 0$)

against

H_1: $\mu_1 > \mu_2$ **or** H_1: $\mu_1 < \mu_2$ **or** H_1: $\mu_1 \neq \mu_2$

at a significance level of α with $\mathrm{DF} = \text{the smaller number of } \{(n_1 - 1) \text{ and } (n_2 - 1)\}$

Requirements

1. A random sample of each population is taken . The sample mean for the sample of one population is s_1 and the sample mean for the sample of a second population is s_2
2. The two samples are **independent** of each other.
3. **Both populations are normal** **or** **both sample sizes are greater than 30.**

Notation for the Samples of Two Population Means

Population 1

μ_1 = population mean

n_1 = sample size

$\bar{x}_1$ = sample Mean from Population 1

s_1 = Sample Standard Deviation from Population 1

Population 2

μ_2 = population mean

n_2 = sample size

$\bar{x}_2$ = Sample Mean from Population 2

s_2 = Sample Standard Deviation from Population 2

Use the **smallest** Degrees of Freedom from Sample 1 and Sample 2.

The Test Statistic for **Two Population Proportions with** H_0: $p_1 = p_2$

Test Statistic: $$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

Testing a Claim about the Difference in 2 Population Means (Independent Samples)

Example 1 (Right Tail Test)

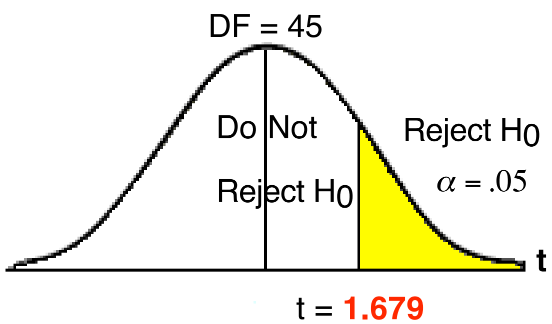
A study was conducted to see if a two week instructional course helped fifteen year olds score better on their first written driving exam. A independent random sample was taken of 46 students at Folsom High School who were given the two week program and then given the written exam. The average exam score for these 46 students was 72.5 points with a standard deviation of 12.3 points. A independent random sample was taken of 66 students at Folsom High School who were not given the course before they were given the written exam. The average exam score for this group of 66 students was 70.2 points with a standard deviation of 10.7 points. Use a $\alpha = .05$ significance level to test the claim that the **students given the course** will have a **higher average exam score** than students **who do not take the course**.

Use the sample with the **highest mean as Sample 1** and the **other sample as Sample 2** to set up H_0 and H_1.

	Sample 1 (given the course)	Sample 2 (not given the course)
H_0: $\mu_1 = \mu_2$	$n = 46$	$n_2 = 66$
H_1: $\mu_1 > \mu_2$	$\bar{x}_1 = 72.5$	$\bar{x}_2 = 70.2$
$\alpha = .05$	$s_1 = 12.3$	$s_2 = 10.7$

Use the smallest Degrees of Freedom from Sample 1 and Sample 2.

Right Tail Test of H_0: $\mu_1 = \mu_2$



Test Statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

$$t = \frac{(72.5 - 70.2)}{\sqrt{\frac{(12.3)^2}{46} + \frac{(10.7)^2}{66}}}$$

$$t = 1.03$$

Conclusion based on H_0 **: Do not Reject H0**

Conclusion based on the problem:

There is not sufficient evidence at the $\alpha = .05$ level to reject the hypothesis that the group given the course would have the same test average as the group not given the course.

or

There is not sufficient evidence at the $\alpha = .05$ level to support the claim that the group given the course would have a higher test average than the group not given the course.

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
45	1.301	1.679	2.014	2.412	2.690

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Example 2 (Left Tail Test)

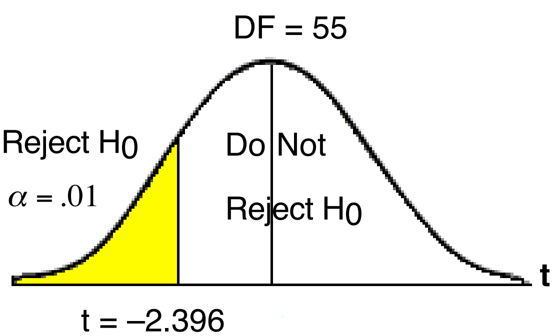
A researcher wants to conduct a study to see if the use of Pine Bark pills can reduce the average number of days a subject is absent from work in a one year period. A random sample of 81 people were given regular treatments of Pine Bark. This group had an one year average number of sick days of 8.3 days with a standard deviation of 1.4 days. A independent random sample of 56 people were given regular treatments of a placebo. This group had an one year average number of sick days of 9.1 days with a standard deviation of 1.5 days. Use a $\alpha = .01$ significance level to test the claim that treatment **with Pine Bark will lower the average number of days a person is absent** from work in a one year period **compared to not using Pine Bark.**

I am selecting the sample treated with Pine Bark as Sample 1 and the placebo group as Sample 2.

	Sample 1 (given Pine Bark)	Sample 2 (given placebo)
H_0: $\mu_1 = \mu_2$		
H_1: $\mu_1 < \mu_2$	$n = 81$	$n_2 = 56$
	$\bar{x}_1 = 8.3$	$\bar{x}_2 = 9.1$
$\alpha = .01$	$s_1 = 1.4$	$s_2 = 1.5$

Use the smallest Degrees of Freedom from Sample 1 and Sample 2.

Right Tail Test of H_0: $\mu_1 = \mu_2$



Test Statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

$$t = \frac{(8.3 - 9.1)}{\sqrt{\frac{(1.4)^2}{81} + \frac{(1.5)^2}{56}}}$$

$$t = -3.15$$

Conclusion based on H_0 **: Reject H0**

Conclusion based on the problem:

There is sufficient evidence at the $\alpha = .01$ level **to support the claim** that treatment with Pine Bark will lower the average number of days a person is absent from work in a one year period.

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
55	1.297	1.673	2.004	2.396	2.668

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Example 3 (Two Tail Test)

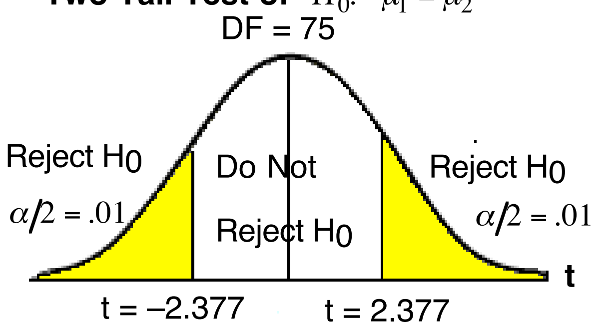
A leading herb company wants to test the effect of a new herbal sleeping aide on the time it takes to go to sleep at night. 101 patients were selected at random to use the herbal sleeping aide. The average time to get to sleep with the herbal aide was 32.6 minutes with a standard deviation of 4.6 minutes. 76 students were selected at random to take a placebo with no herbal aide. This groups average time to get to sleep was 34.8 minutes with a standard deviation of 5.2 minutes. Use a .02 significance level to test the claim that there is **no difference in the average time needed to get to sleep** between those who use the herb and those that do not use the herb.

Select the sample treated with Pine Bark as Sample 1 and the placebo group to be Sample 2.

	Sample 1 (given Herb)	Sample 2 (not given herb)
H_0: $\mu_1 = \mu_2$		
H_1: $\mu_1 \neq \mu_2$	$n = 101$	$n_2 = 76$
	$\bar{x}_1 = 32.6$	$\bar{x}_2 = 34.8$
$\alpha = .02$ so $\alpha/2 = .01$	$s_1 = 4.6$	$s_2 = 5.2$

Use the **smallest** Degrees of Freedom from Sample 1 and Sample 2.

Two Tail Test of H_0: $\mu_1 = \mu_2$



Test Statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

$$t = \frac{(32.6 - 34.8)}{\sqrt{\frac{4.6^2}{101} + \frac{5.2^2}{76}}}$$

$$t = -2.93$$

Conclusion based on H_0 **: Reject H0**

Conclusion based on the problem:

There is sufficient evidence at the $\alpha = .02$ significance level **to reject the hull hypothesis that there is no difference** in the time needed to get to sleep between those who use the herb and those that do not.

or

There is sufficient evidence at the $\alpha = .02$ level **to support the claim that there is a difference** in the time needed to get to sleep between those who use the herb and those that do not.

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
75	1.293	1.665	1.992	2.377	2.643

