

Finding Critical Values for a Standard Normal Z Distribution

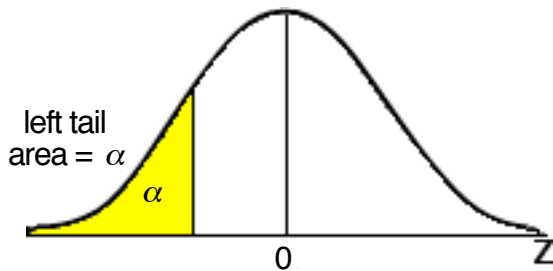
A **Critical Value for a Standard Normal Distribution** is a **z value** on the z axis that is the vertical boundary **separating the area in one tail of the graph from the remaining area.**

The **total tail area is denoted by α .**

In this chapter the tail areas are normally .10 or less
so the tail areas are shown as a smaller proportion of the total area.

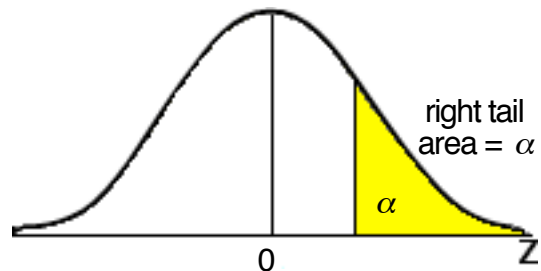
Left Tail (One Tail)

The total value of α is placed
in the **left tail area**



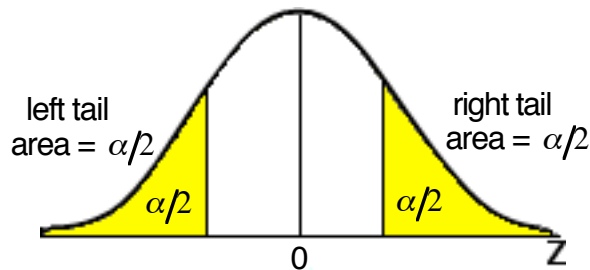
Right Tail (One Tail)

The total value of α is placed
in the **right tail area**



Two Tail (Left and Right Tails)

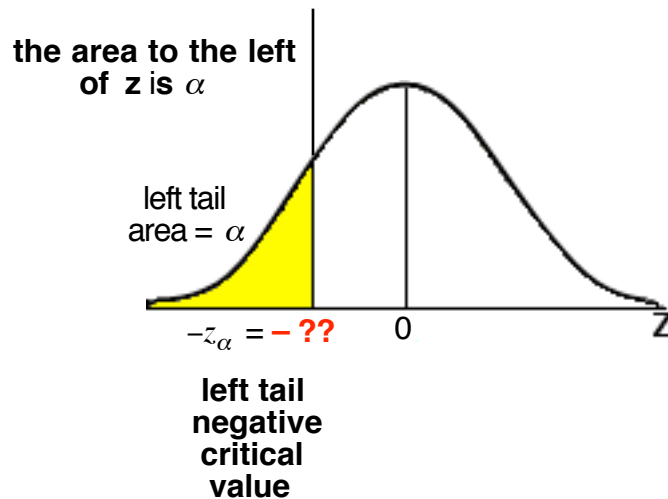
The total value of α is split in half
and an area of $\alpha/2$ is placed in **both** the **left tail** and the **right tail.**



**Finding the Left Tail (One Tail) Critical Value
for a Standard Normal Distribution
with a left tail area of α**

In the case of a Left Tail (One Tail) example the entire value of α is placed in the left tail. There is no right tail critical area. **The total area denoted by α is in the left tail.**

We call the z value that separates a left tail area of α from the rest of the area under the curve a **Left Tail Critical Value**. The left tail has a Negative Critical Z Value that has an area of α to the left. The **Left Tail Critical Value for a left tail area of α is written $-z_\alpha$**



Negative Z Scores table

The numbers in the body of the table (in yellow)
represents an area to the LEFT of a given z score

The z score is the 2 digit z value from the left column (in red)
with an additional decimal place from the row on top (in red)

A number in yellow that is at the
intersection of the left column z value (in red) and top row z value (in red)
stands for the area to the **LEFT** of that given z score

The Negative Z Scores Table is used to find the **Left Tail Critical Value** if the area denoted by α is in the left tail. The number in YELLOW at the **intersection of the left column (in red) and top row (in red)** stands for the **area to the LEFT** of that given z score

Find a number in yellow in the body of the table as close to α as possible. The **left tail critical z value** is the 2 digit z value from the left column (in red) with an additional decimal place from the row on top (in red).

Negative Z Scores										
Standard Normal (Z) Distribution: Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
This is only a portion of the entire Negative z Score Table										

**Finding The Left Tail (One Tail) Critical Value for the z Distribution
with a left tail area of α**

Example 1

Find the Left Tail (One Tail) Critical Value for an area of $\alpha = .10$ in the left tail

The number at the intersection of the left column (in red) and top row (in red) stands for the area to the **LEFT** of that given z score **Find a number in YELLOW in the body of the table as close to .10 as possible**. The Left Tail critical z value is the 2 digit z value from the left column (in red) with an additional decimal place from the row on top (in red).

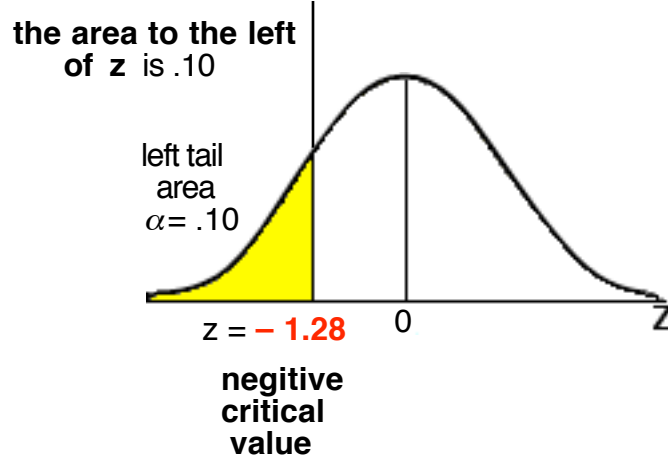
Find a number in yellow in the body of the table as close to .10 as possible

The number at the intersection of the **-1.2** row and the **0.08** column is **0.1003**
which is as close to .10 as possible

This means that the Z score with a left tail area of .10
is **Z = -1.28**

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985

The Left Tail critical z value



**Finding The Left Tail (One Tail) Critical Value for the z Distribution
with a left tail area of α**

Example 2

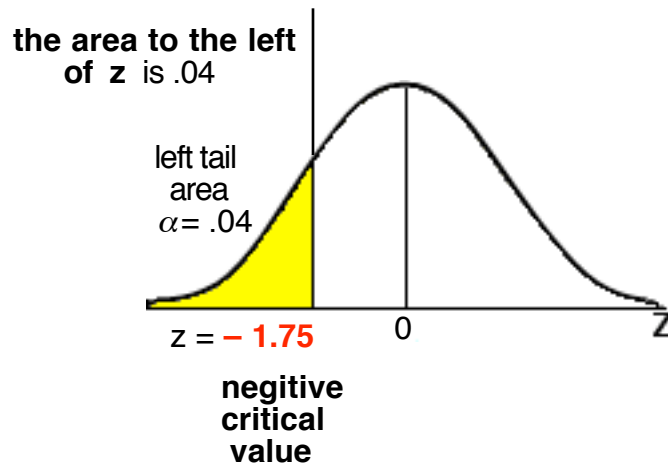
Find the Critical Value for an area of $\alpha = .04$ in the left tail

Find a number in **YELLOW** in the body of the table as close to .04 as possible

The number at the intersection of the **-1.7** row and the **0.05** column is **0.0401**
which is as close to .04 as possible

This means that the Z score with a left tail area of .04
is **Z = -1.75**

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367



**Finding The Left Tail (One Tail) Critical Value for the z Distribution
with a left tail area of α**

Example 3

Find the Critical Value for an area of $\alpha = .05$ in the left tail

Find a number in **YELLOW** in the body of the table as close to .05 as possible

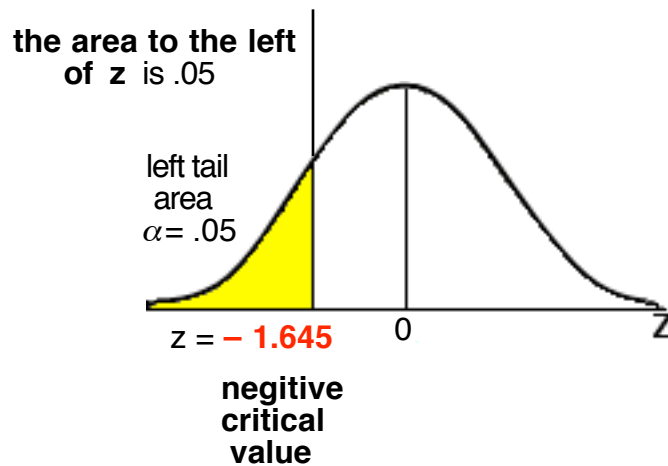
The body of the table has a **0.0505** and a **0.0495** which are equally close to .0500

but

The cells at the bottom of the table states that for an area of **0.0500** use $Z = -1.645$

$Z = -1.645$

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
Z scores	of -3.5 or less use .0001				AREA	Z Score		AREA	Z Score	
					0.0500	-1.645		0.0050	-2.575	



The Right Tail Critical Value for the z Distribution with a right tail area of α

A **Right Tail Critical z Value** for a Standard Normal Distribution is a **positive z value** on the z axis that is the vertical boundary separating **the area in the right tail denoted by α** from the area to the left of the **Critical z Value**.

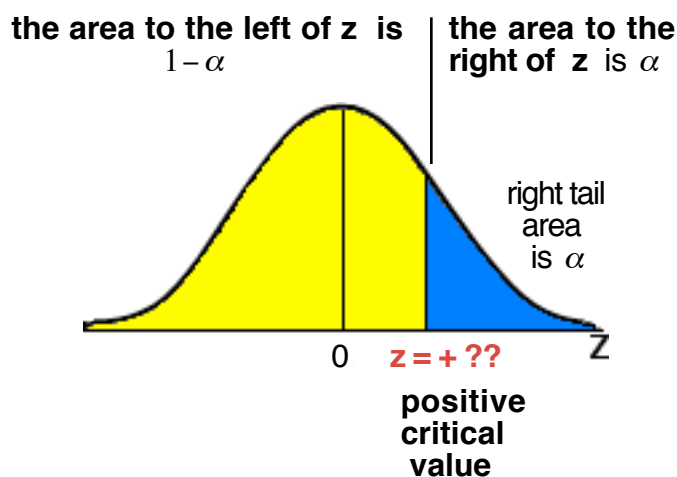
The Positive Z Scores Table is used to find the **Right Tail Critical z Value** if the area denoted by α is in the right tail. The use of the positive z table requires an additional step. We are trying to find a z score **given an area to the right of the z score but the z tables lists the area to the left of the z score**. When we are given a right tail area of α we must **first find the area to the left of z** by subtracting the right tail area α from 1. If the right tail area is α then area to the left of z is $1 - \alpha$. We then use the Positive z Table to look for an area of $1 - \alpha$ to the left of the z value.

Finding the Right Tail Critical Value for the z Distribution with a right tail area of α

The area to the right of the z value is given as α

The area to the left of the z value is $1 - \alpha$

The critical value is the positive z score that has an area of $1 - \alpha$ to the left of the z value.



The Positive Z Scores Table is used to find the **Right Tail Z Value** if the area denoted by α is in the right tail. The area to the right of the **Right Tail Z Value** is shown in the right tail as α **and the area to the left of the z Value** is shown to the left of the z value as $1 - \alpha$

Positive Z Scores Table

The numbers in **YELLOW** in the body of the table represent an area **to the left** of a given z score the numbers in the table are greater than .5000 because the right tail area is very small so the area in the left tail is large

The z score is the 2 digit z value from the left column (in red) with an additional decimal place from the row on top (in red)

A number in yellow that is at the intersection of the left column z value (in red) and top row z value (in red) stands for the area to the **LEFT** of that given z score

Find a number in the body of the table as close to α as possible (in yellow). The **right tail critical z value** is the 2 digit z value from the left column (in red) with an additional decimal place from the row on top (in red).

Positive Z Scores										
Standard Normal (Z) Distribution: Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
This is only a portion of the entire Positive z Score table										

Finding the Right Tail (One Tail) Critical Value for the z Distribution with a right tail area of α

Example 4

Find the Critical Value for an area of $\alpha = .08$ in the right tail

The Z Tables only give areas to the left of a Z value.

If we are given an area to the right of a z value
then we must find the remaining area that is to the left of the Z value
and then look that area up in the Z table

If the area to the right of the Z score is .08
then the area to the left of the Z score is $1 - .08 = .9200$

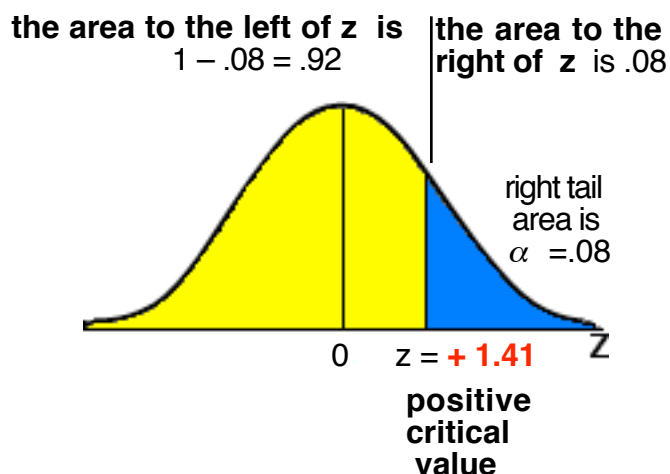
Find a number in YELLOW in the body of the table as close to .9200 as possible

The number at the intersection of the **1.4** row and the **0.01** column is **0.9070**

which is as close to .9200 as possible

$$Z = 1.41$$

Positive Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319



Finding the Right Tail (One Tail) Critical Value for the z Distribution with a right tail area of α

Example 5

Find the Critical Value for an area of $\alpha = .01$ in the right tail

The Z Tables only give areas to the left of a Z value.

If we are given an area to the right of a z value
then we must find the remaining area that is to the left of the Z value
and then look that area up in the Z table

If the area to the right of the Z score is .01
then the area to the left of the Z score is $1 - .01 = .9900$

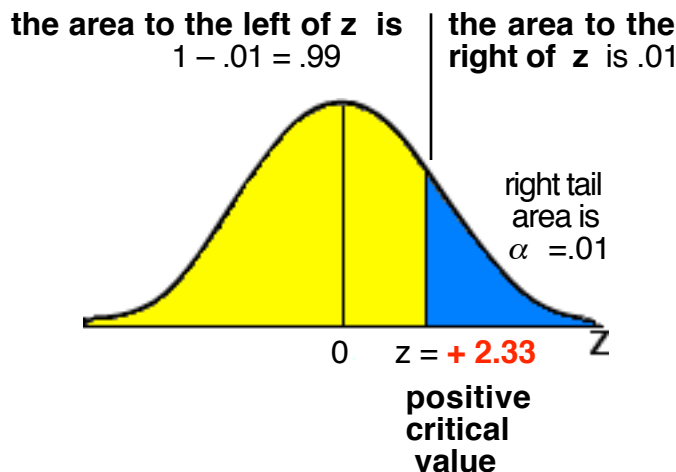
Find a number in YELLOW in the body of the table as close to .9900 as possible

The number at the intersection of the **-2.3** row and the **0.03** column is **0.9901**

which is as close to .9900 as possible

This means that the Z score with a right tail area of .01
is **Z = 2.33**

Positive Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916



**Finding the Right Tail (One Tail) Critical Value for the z Distribution
with a right tail area of α**

Example 6

Find the Critical Value for an area of $\alpha = .05$ in the **right tail**

The Z Tables only give areas to the left of a Z value.

If we are given an area to the **right** of a z value
then we must find the remaining area that is to the **left** of the Z value
and then look that area up in the Z table

If the area to the right of the Z score is .05
then the area to the **left** of the Z score is $1 - .05 = .9500$

Find a number in **YELLOW** in the body of the table as close to .9500 as possible

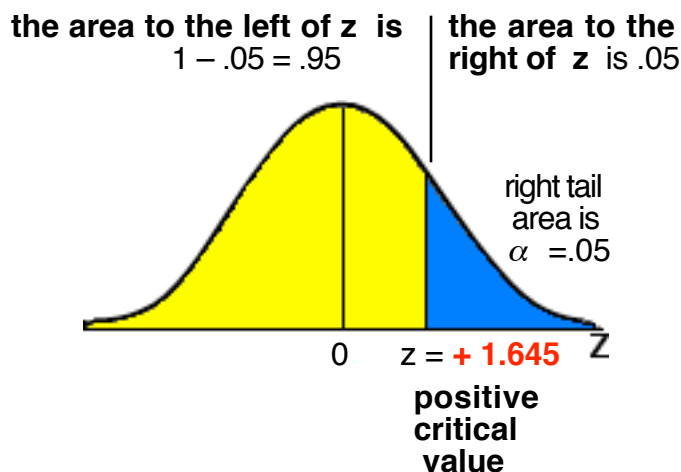
The body of the table has a **0.9495** and a **0.9505** which are equally close to .9500

but

The cells at the bottom of the table show that for an area of **0.9500** use **Z = 1.645**

Z = 1.645

Positive Z Scores										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
Z scores of 3.5 or more use .9999					AREA	Z Score		AREA	Z Score	
					0.9500	1.645		0.9950	2.575	



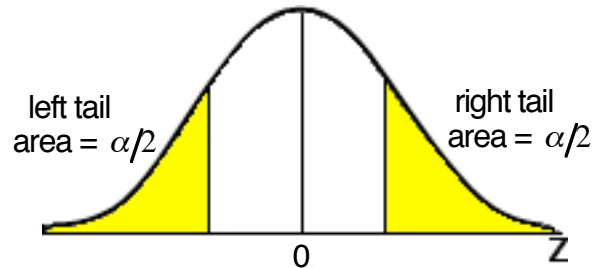
Finding Left and Right Tail Critical Values (Two Tail) for a Standard Normal Distribution Z

A **Critical Value** for a Standard Normal Distribution is a **z value on the z axis** that is the vertical boundary separating the area in one tail of the graph from the remaining area.

The **total tail area in both tails is denoted by α**
 In this chapter the tail areas are normally .10 or less
 so the tail areas are shown as a small proportion of the total area

Left and Right tail areas

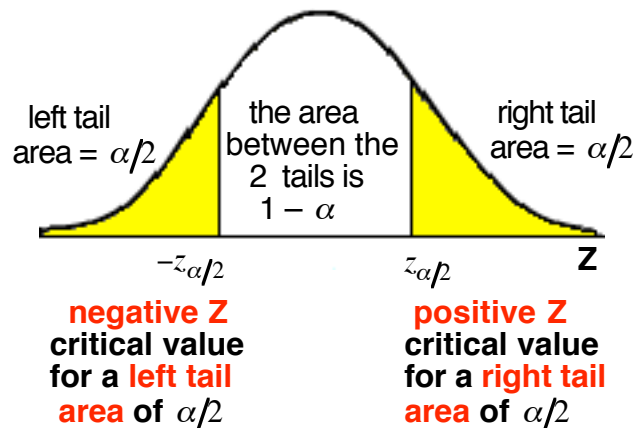
The **total area in both tails is α**
 if this total area is **divided equally between the left and right tails**
 then **the left and right tail will each have an area of $\alpha/2$**



How are $-z_{\alpha/2}$ and $+z_{\alpha/2}$ related

The positive critical value $+z_{\alpha/2}$ is equal to $|-z_{\alpha/2}|$

the total area for both tails is α



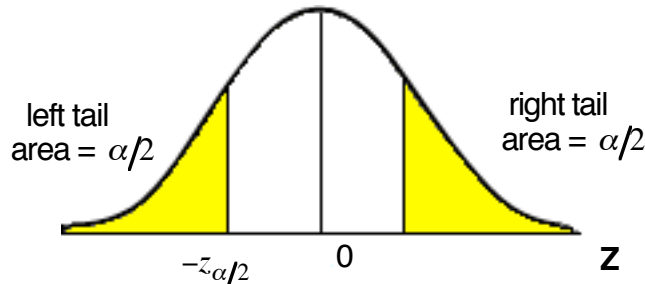
Critical Values

The Left Tail Critical Value $-z_{\alpha/2}$

for a Standard Normal Distribution Z if the area to the left of $z_{\alpha/2}$ is $\alpha/2$

We call the z value that separates a left tail area of $\alpha/2$ from the rest of the area under the curve a **Left Tail Critical Value**. The left tail area of $\alpha/2$ must have a negative z value that has an area of $\alpha/2$ to its left. The **Left Tail Critical Value for a left tail area of $\alpha/2$ is written $-z_{\alpha/2}$**

the total area for both tails is α



**left tail
critical value**

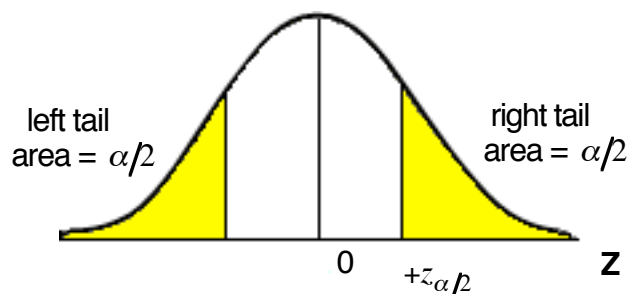
the area to the left
of $-z_{\alpha/2}$ is $\alpha/2$

The Right Tail Critical Value $z_{\alpha/2}$

for a Standard Normal Distribution Z if the area to the right of $z_{\alpha/2}$ is $\alpha/2$

We call the z value that separates a right tail area of $\alpha/2$ from the rest of the area under the curve a **Right Tail Critical Value**. The right tail area of $\alpha/2$ must have a positive z value that has an area of $\alpha/2$ to its right. The **Right Tail Critical Value for a right tail area of $\alpha/2$ is written $z_{\alpha/2}$ or $+z_{\alpha/2}$**

the total area for both tails is α



**right tail
critical value**

the area to the right
of $z_{\alpha/2}$ is $\alpha/2$

Finding Left and Right Tail Critical Values (Two Tail) for a Standard Normal Distribution Z

Example 7

The total area for both tails is $\alpha = .05$ If $\alpha = .05$ then $\alpha/2 = .025$

Find the **Left Tail Critical Value** $-z_{\alpha/2}$ with a **left tail area of** $\alpha/2 = .025$

The Negative Z Scores Table is used to find the z score with an area of .05 to its left.

Find an number in **YELLOW** in the body of the table as close to .025 as possible

The number at the intersection of the **-1.9** row and the **.06** column is 0.0250

The tail area to the left of $z = -1.96$ is .025

$$-z_{\alpha/2} = -1.96$$

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233

Find the **Right Tail Critical Value** $z_{\alpha/2}$ with a **right tail area of** $\alpha/2 = .025$

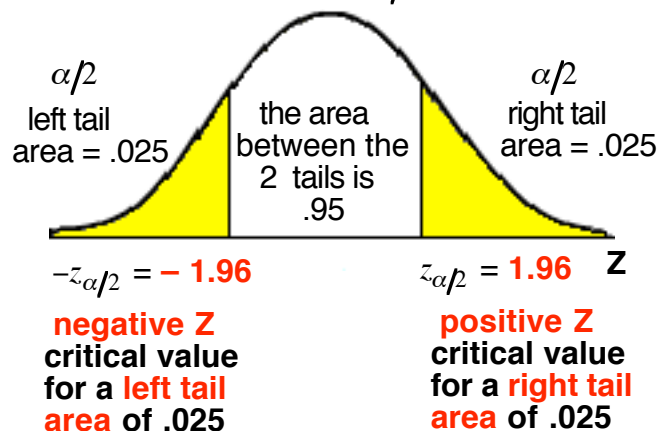
The negative critical value $-z_{\alpha/2}$ is **-1.96**

The positive critical value $+z_{\alpha/2}$ is equal to $|-z_{\alpha/2}|$

The positive critical value $+z_{\alpha/2}$ is **1.96**

the total area for both tails is $\alpha = .05$

If $\alpha = .05$ then $\alpha/2 = .025$



Finding Left and Right Tail Critical Values (Two Tail) for a Standard Normal Distribution Z

Example 8

The total area for both tails is $\alpha = .08$ If $\alpha = .08$ then $\alpha/2 = .04$

Find the **Left Tail Critical Value** $-z_{\alpha/2}$ with a **left tail area of** $\alpha/2 = .04$

The Negative Z Scores Table is used to find the z score with an area of .04 to its left.

Find a number in **YELLOW** in the body of the table as close to .0400 as possible

The body of the table has a .0401 which is **as close to .0400 as possible**

the number at the intersection of the **-1.7** row and the **.05** column is 0.0401

$$-z_{\alpha/2} = -1.75$$

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367

Find the **Right Tail Critical Value** $z_{\alpha/2}$ with a **right tail area of** $\alpha/2 = .04$

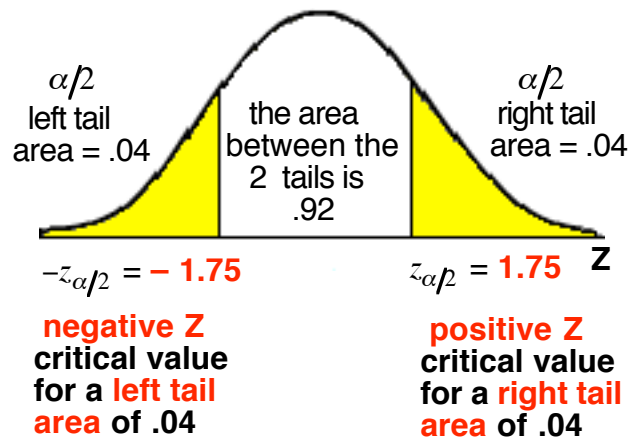
The negative critical value $-z_{\alpha/2}$ is **-1.75**

The positive critical value $+z_{\alpha/2}$ is equal to $|-z_{\alpha/2}|$

The positive critical value $+z_{\alpha/2}$ is **1.75**

the total area for both tails is $\alpha = .08$

If $\alpha = .08$ then $\alpha/2 = .04$



Finding Left and Right Tail Critical Values (Two Tail) for a Standard Normal Distribution Z

Example 9

The total area for both tails is $\alpha = .10$ If $\alpha = .10$ then $\alpha/2 = .05$

Find the **Left Tail Critical Value** $-z_{\alpha/2}$ with a **left tail area of** $\alpha/2 = .05$

The Negative Z Scores Table is used to find the z score with an area of .05 to its left.

Find an number in **YELLOW** in the body of the table as close to .05 as possible

The cells at the bottom of the z table say to use **Z = - 1.645** for a left tail area of **0.0500**

$$-z_{\alpha/2} = - 1.645$$

Negative Z Scores							
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z							
Z scores	of -3.5 or less use .0001		AREA	Z Score		AREA	Z Score
			0.0500	-1.645		0.0050	-2.575

Find the **Right Tail Critical Value** $z_{\alpha/2}$ with a **right tail area of** $\alpha/2 = .05$

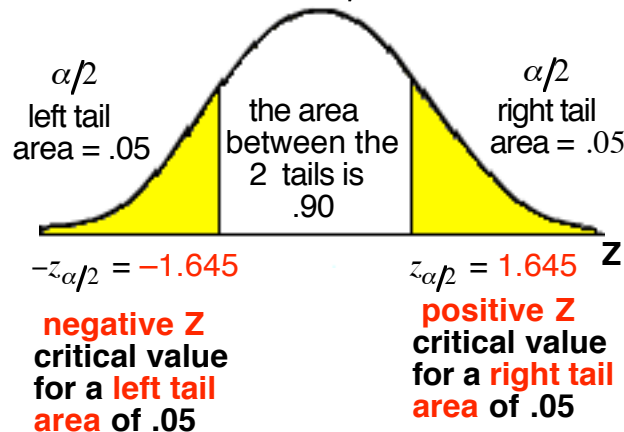
The negative critical value $-z_{\alpha/2}$ is **- 1.645**

The positive critical value $+z_{\alpha/2}$ is equal to $|-z_{\alpha/2}|$

The positive critical value $+z_{\alpha/2}$ is **1.645**

the total area for both tails is .10

If $\alpha = .10$ then $\alpha/2 = .05$



Finding Left and Right Tail Critical Values (Two Tail) for a Standard Normal Distribution Z

Example 10

The total area for both tails is $\alpha = .01$ If $\alpha = .01$ then $\alpha/2 = .005$

Find the **Left Tail Critical Value** $-z_{\alpha/2}$ with a **left tail area of** $\alpha/2 = .005$

The Negative Z Scores Table is used to find the z score with an area of .005 to its left.

Find an number in **YELLOW** in the body of the table as close to .005 as possible

The cells **at the bottom of the z table** say to use **Z = - 2.575** for a left tail area of **0.0050**

$$-z_{\alpha/2} = - 2.575$$

Negative Z Scores							
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z							
Z scores	of -3.5 or less use .0001	AREA	Z Score	AREA	Z Score	AREA	Z Score
		0.0500	-1.645	0.0050	-2.575		

Find the **Right Tail Critical Value** $z_{\alpha/2}$ with a **right tail area of** $\alpha/2 = .005$

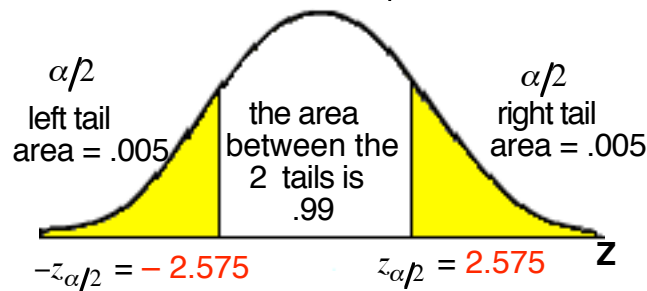
The negative critical value $-z_{\alpha/2}$ is **- 2.575**

The positive critical value $+z_{\alpha/2}$ is equal to $|-z_{\alpha/2}|$

The positive critical value $+z_{\alpha/2}$ is **2.575**

the total area for both tails is $\alpha = .01$

If $\alpha = .01$ then $\alpha/2 = .005$



negative Z
critical value
for a **left tail**
area of .005

positive Z
critical value
for a **right tail**
area of .005