The Techniques of Hypothesis Testing

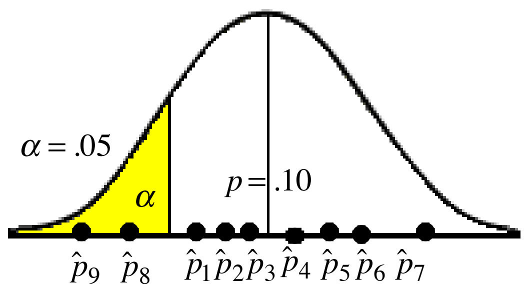
A claim is made that 10% of the population is left handed. An alternate claim is made that less than 10% of the population is left handed. We will use the techniques of hypothesis testing to decide if we will support. the claim that 10% of the population is left handed or not.

H_O: $p = .10$ I think **10% of the population is left handed**.

H_1: $p < .10$ I think that **less than 10% the population is left handed**.

The start of the process is to take a sample and find the proportion of left handed people in the sample. This sample proportion is labeled $\hat{p}$. The **sample proportion** we find is $\hat{p} = .09$ which is just one of many possible values of $\hat{p}$. If we took other samples we would get many different sample proportions. $\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4, \ldots, \hat{p}_n$. The sample proportion is not equal to the claim **H_O:** $p = .10$. This is not unexpected. Even if the population proportion is $p = .10$. we cannot expect a sample proportion equal .10e If the value of the population proportion is in fact **p = .10** than we can expect that **most of the values** of the sample proportions $\hat{p}_1, \hat{p}_2, \hat{p}_3, \ldots, \hat{p}_n$ **will be close to .10**

The distribution below is the distribution of all the possible sample proportions of size n. 9 of the possible sample values are shown. If the claim that p =.10 is true then most of the values form the samples will be close to .10



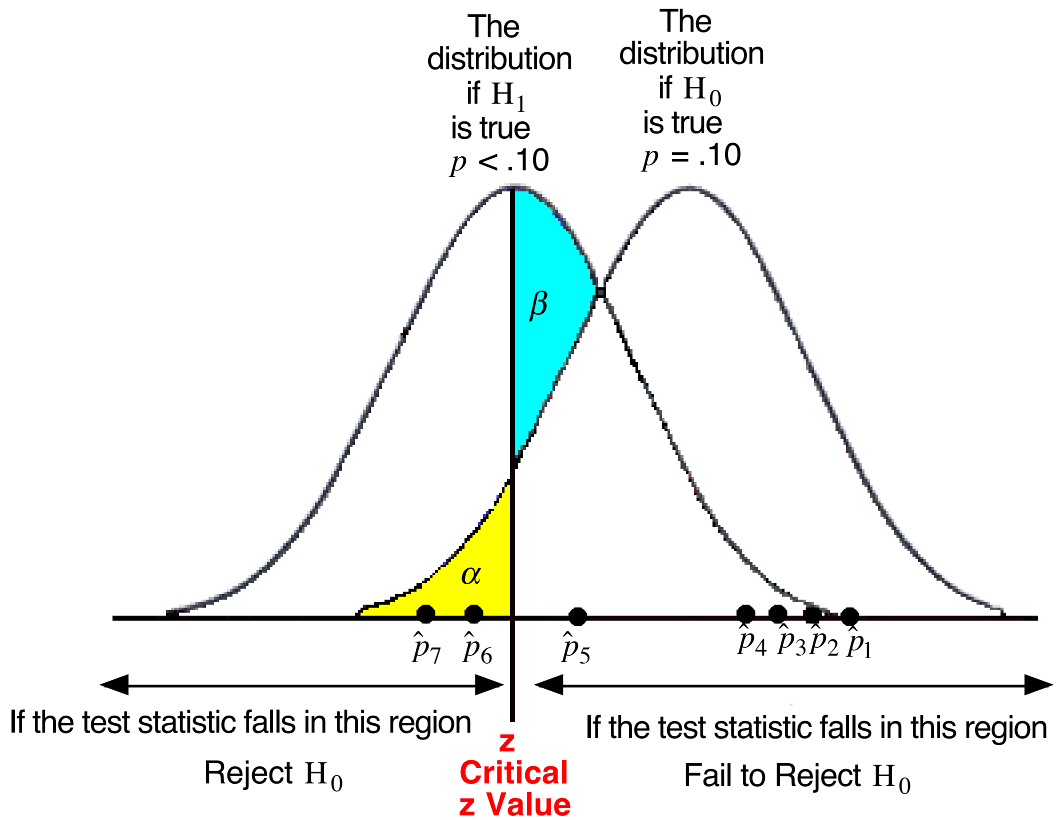
The null hypothesis **H_O:** **p = .10** is tested against the alternate hypothesis **H_1:** **p < .10.**

We use our single sample proportion $\hat{p}$ to test the two hypotheses. A sample value for $\hat{p}$ greater than 10 , like $\hat{p}_4, \hat{p}_5, \hat{p}_6, \hat{p}_7$, would NOT cause us to wonder if **H_O** is false. A sample value for $\hat{p}$ **just a bit less** than .10 , like $\hat{p}_1, \hat{p}_2, \hat{p}_3$, would NOT cause us to wonder if **H_O** is false. A sample value for $\hat{p}$ **a lot less than .10** , like $\hat{p}_8$ or $\hat{p}_9$, **WOULD** cause us to wonder if **H_O** is false. As the value of the sample proportion gets farther to the left of .10 the **less likely that H_O: p = .10 is true and more likely that H_1: p < .10 is true.**

The question is, how far to the left of .10 does the sample value need to be before we **stop accepting H_O: p = .10 and start accepting H_1: p < .10. I should come as no surprise that the number of standard deviations between the sample value and the claim of .10 will be part of the answer for how far to the lest we can go before we reject H_O: p = .10**

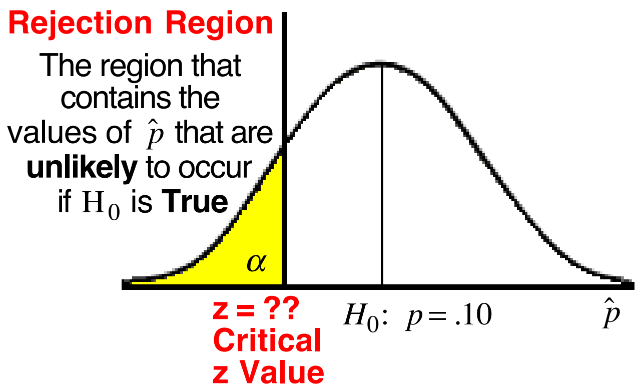
The **Critical Region (or Rejection Region)** and it's **Critical z Value**

In the diagram below, the normal distribution on the **right** is based on **H_O:** **p = .10** being true. The normal distribution on the left is based on **H_1:** **p < .10** being true. If the population proportion p is equal to .10 than most of the possible sample proportions will be close to .10. If our single sample proportion $\hat{p}$ were any of the values $\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4$ than this would support the claim that our that the sample came from a distribution with **p = .10**. This would support **H_O:** **p = .10.**



The location of $\hat{p}_5$ is farther to the left of p = .10 than the first 4 sample proportions are. $\hat{p}_5$ is getting a bit more likely to be from the distribution with **p < .10.** than it is to be from the distribution with **p = .10.** The location of the sample proportions $\hat{p}_6$ and $\hat{p}_7$ are a different matter. The values of $\hat{p}_6$ and $\hat{p}_7$ are in the small left tail of the the distribution with p = .10. The yellow area of the left tail is the probability that a value like $\hat{p}_6$ occurs if p = .10 is true. This is a small probability. The values of $\hat{p}_6$ and $\hat{p}_7$ also fall in the large middle area of the distribution with p < .10. It is much more likely that the sample values $\hat{p}_6$ and $\hat{p}_7$ come from a distribution with **p < .10**. This would support **H_1:** **p < .10**. As the distance between $\hat{p}$ and p grows, the probability that the sample came from a distribution with p = .10 decreases. The distance that $\hat{p}$ can be to the left of p = .10 before you stop supporting the claim p = .10 and start supporting p < .10 is called the **Critical z Value.** The **Critical z Value** is the maximum number of standard deviations that $\hat{p}$ can be away from p before we reject **H_O:** **p = .10** and support the alternate **H_1:** **p < .10**. The yellow region in the left tail that contains the z values beyond the **Critical z Value** is called the **Rejection Region**. The area of the critical region is α, the value for the level of significance.

The values of $\hat{p}$ that are **unlikely to occur** if **H_O: p = .10** is true are found in the **left tail area.** The region that contains these **unlikely** values is called the **Rejection Region**. The area of the rejection region is α , the value for the level of significance. A **Critical Value** is **a value on the z axis** that is the boundary separating rejection region from the rest of the area under the curve. The **Critical z Value** is the maximum number of standard deviations that $\hat{p}$ can be away from p before we Reject The Null Hypothesis **H_O: p = .10** and support the Alternate Hypothesis **H_1: p < .10.**



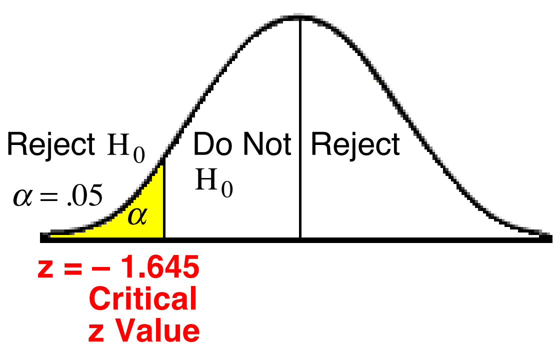
The **Critical z Value** is found by determining the z score for a **left tail area of size** α**.**

Find the Critical Z Value for a left tail area of .05

If the area in the left tail is α = .05 then we find the **critical z Value** by determining the z score for a **left tail area of** .05

Negative Z Scores										
Z scores	of –3.5 or less use .0001				AREA	Z Score		AREA	Z Score	
					0.0500	–1.645		0.0050	–2.575	

The z score with an area to the left of .05 is **z = –.1645** is .05 Place the Critical Z Value on the z axis at **z = –.1645.** This is on the z axis where the yellow area in the left tails ends. Label the left tail area as the **Reject H_O** region. Label the right tail area in white as the **DO Not Reject H_O** region.



The z Test Statistic for the **Population Proportion**

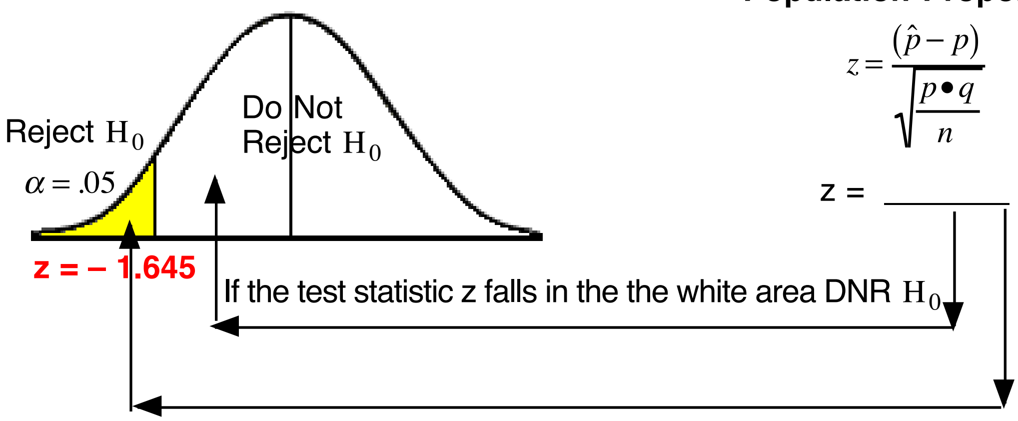
The z Test Statistic is a value that represents the **number of standard deviations that $\hat{p}$ is away from p.** If the z Test Statistic for $\hat{p}$ is to the left of the **Critical z Value** for a left tail area of α then we will **Reject the hypothesis H_0: p = .10** and support the Alternate Hypothesis **H_1: p < .10**. If the z Test Statistic for $\hat{p}$ is to the right of the **Critical z Value** for a left tail area of α then we will **Not Reject the hypothesis H_0: p = .10**

Compute the Test **Statistic** for The **Population Proportion**
determine if that value falls to the left of the Critical z value.

The **z Test Statistic** for the
Population Proportion

$$z = \frac{(\hat{p} - p)}{\sqrt{\frac{p \bullet q}{n}}}$$

z = ________



If the z test statistic falls the the yellow area Reject H_0

Locate the value of the value of the test statistic on the z axis. The test statistic is the number of standard deviations that $\hat{p} = .09$ is away from $p = .10$ If the number of standard deviations is less than the Critical Value of **z = – 1.645** we **DO Not Reject H_o** . If the number of standard deviations is more than the Critical value of **z = – 1.645** we **Reject H_o**.

The conclusion of an Hypothesis is either **Reject H_o** or **DO Not Reject H_o.**

After we state our conclusion as **RH_o** or **DNR H_o** we need to write an English sentence that describes the conclusion based on the wording in the actual problem,

Reject H_o

We conclude that there is sufficient evidence at the α level to support the H_1 claim that " ...."

Do Not Reject H_o

We conclude that there is **not** sufficient evidence at the α level to **reject** the **H_o** hypothesis that "......"
or
There is **not** sufficient evidence at the α level to **accept** the **H_1** claim that " ...."

An Hypotheses Test for an

H_1 statement that makes a LESS THAN claim

H_1: $p < .10$

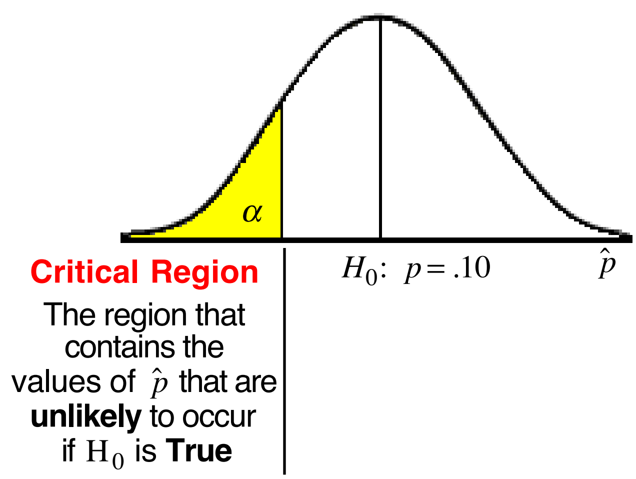
One Tail (Left Tail) Critical Value

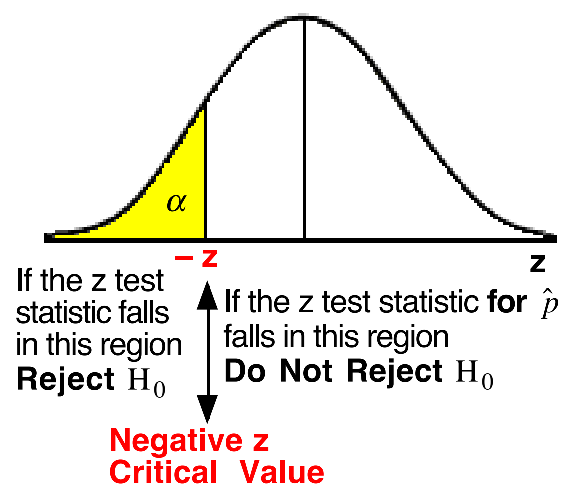
If the Alternate Hypothesis **H_1** is a **LESS THAN** statement like **H_1: $p < .10$** then the small values of $\hat{p}$ that are **unlikely to occur** if **p = .10** is true are found in the **left tail area**. The region that contains these **unlikely small** values is called the **Left Tail Rejection Region**. The area of the rejection region is α, the value for the level of significance.

The z score that is the boundary for the **rejection Region** is called the **Negative Critical z Value** for the **left tail area of** α. The **Critical z Value** is the maximum number of standard deviations that $\hat{p}$ can be to the **left** of p before we **Reject The Null Hypothesis H_0: p = .10** and support and **support the Alternate Hypothesis H_1: p < .10.**

H_0: $p = .10$ I think that **the proportion of the population that is left handed IS EQUAL TO 10%.** .

H_1: $p < .10$ I think that **the proportion of the population that is left handed IS LESS THAN 10%.**





The **z Test Statistic** for a given $\hat{p}$ and a value of p stated in **H_0** is $z = \dfrac{(\hat{p} - p)}{\sqrt{\dfrac{p \bullet q}{n}}}$

An Hypotheses Test for an

H_1 statement that makes a GREATER THAN claim

H_1: $p > .10$

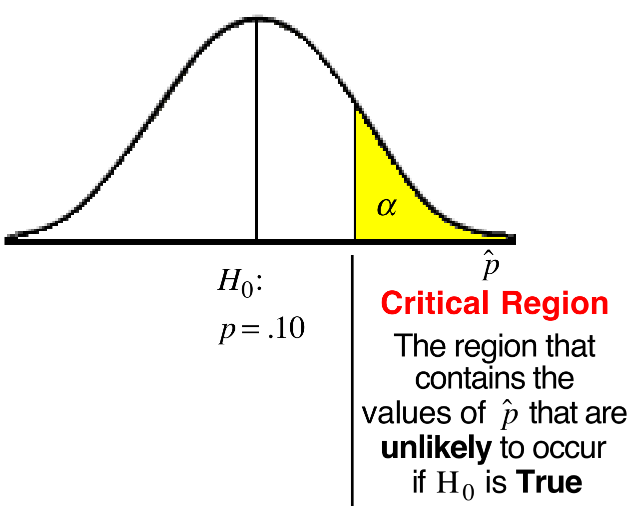
One Tail (Right Tail) Critical Value

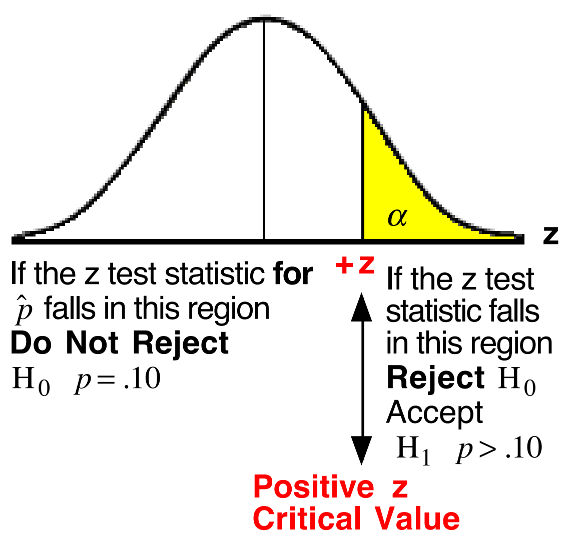
If the Alternate Hypothesis **H_1** is a greater than statement like **H_1: $p > .10$.** then the large values of $\hat{p}$ that are **unlikely to occur** if **p = .10** is true are found in the **right tail area**. The region that contains these **unlikely large** values is called the **Right Tail Critical Region**. The area of the critical region is α, the value for the level of significance.

The z score that is the boundary for the **Right Critical Region** is called the **Positive Critical z Value** for the **right tail area of** α. The **Critical z Value** is the maximum number of standard deviations that $\hat{p}$ can be to the **right** of p before we **Reject The Null Hypothesis H_0: p = .10** and support and **support the Alternate Hypothesis H_1: p >.10.**

H_0: $p = .10$ I think that **the proportion of the population that is left handed IS EQUAL TO 10%.** .

H_1: $p > .10$ I think that **the proportion of the population that is left handed IS MORE THAN 10%.**





The **z Test Statistic** for a given $\hat{p}$ and a value of p stated in the **H_0** is $z = \dfrac{(\hat{p} - p)}{\sqrt{\dfrac{p \bullet q}{n}}}$

An Hypotheses Test for an
H_1 statement that makes a NOT EQUAL to claim
H_1: $p \neq .10$

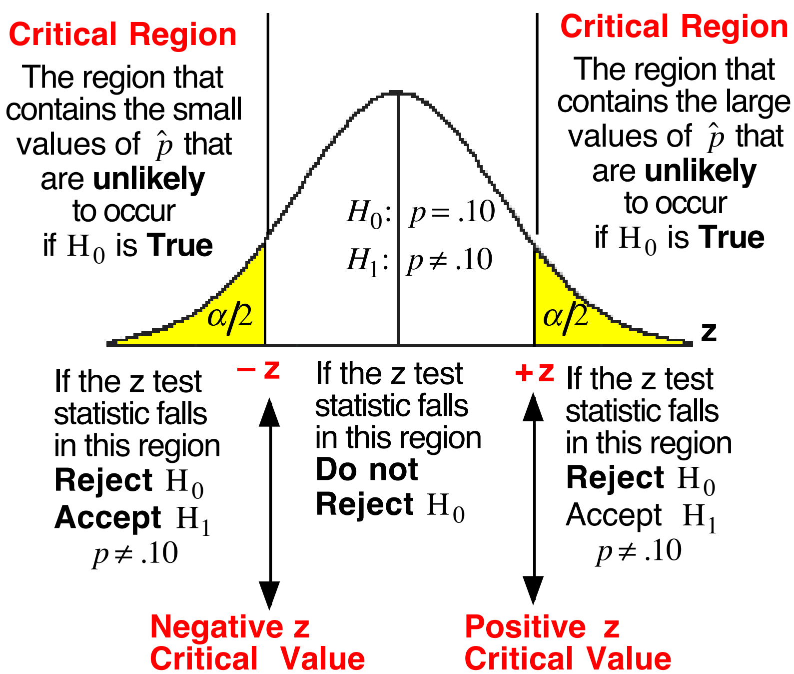
Two Tail (Left and Right Tails) Critical Values

If the Alternate Hypothesis H_1 is a **not equal to statement** like H_1: **$p \neq .10$.** Then the small values of $\hat{p}$ that are **unlikely to occur** if **p = .10** is true are found the in the **left tail area and** the large values of $\hat{p}$ that are **unlikely to occur** if **p = .10** is true are found the in the **right tail area**. There are 2 regions that contains these **unlikely** values. The area of each of the the critical regions is $\alpha / 2$.

The z score that is the boundary for the **Left Critical Region** is called the **Negative Critical z Value** for the **left tail area of** $\alpha / 2$. The z score that is the boundary for the **Right Critical Region** is called the **Positive Critical z Value** for the **right tail area of** $\alpha / 2$. The **Critical z Value** is the maximum number of standard deviations that $\hat{p}$ can be away from p before we **Reject The Null Hypothesis** H_0: **p = .10** and support and **support the Alternate Hypothesis** H_1: **p >.10.**

H_0: $p = .10$ I think that **the proportion of the population that is left handed IS EQUAL TO 10%.** .

H_1: $p \neq .10$ I think that **the proportion of the population that is left handed IS NOT EQUAL TO 10%.**



The **z Test Statistic** for a given $\hat{p}$ and a value of p stated in H_0 is $z = \dfrac{(\hat{p} - p)}{\sqrt{\dfrac{p \bullet q}{n}}}$

