Section 8 – 1A: An Introduction to Hypothesis Testing

The Purpose of Hypothesis Testing

See's Candy states that a box of it's candy weighs 16 oz. They do not mean that every single box weights **exactly** 16 oz. Some of the boxes would weigh less than 16 oz. and some of the boxes would weigh exactly 16 oz. and some would weigh more than 16 oz.

What See's really means is that the **average weight of all the boxes** would be **equal to 16 oz.**

See's is saying that the **Population Mean is equal to 16 oz**. $\mu_x = 16$

We do not know if See's claim that $\mu_x = 16$ is true.

We call this statement $\mu_x = 16$ a **Null Hypothesis**

The **null hypothesis H_O** is always an **equality statement** about a **population parameter.**

The null hypothesis H_O contains an equal sign.

A customer claims that the **average weight** of all the boxes is **less than 16 oz.**
The customer is saying that the **Population Mean is less than 16 oz**, $\mu_x < 16$

We do not know if the customer's claim that $\mu_x < 16$ is true.

We call the statement $\mu_x < 16$ an **Alternate Hypothesis**

The **Alternate Hypothesis H_1** is an **inequality that disagrees with the Null Hypothesis H_O**

The purpose of Hypothesis Testing is to
decide which of the two hypothesis to support.

The company states a **Null Hypothesis** that the **Population Mean is equal to 16 oz**, $\mu_x = 16$

H_O: $\mu_x = 16$

The customer states an **alternate hypothesis** that the **Population Mean is less than 16 oz**,

H_1: $\mu_x < 16$

You must decide which of the two statements to support

H_O: $\mu_x = 16$ **or** **H_1:** $\mu_x < 16$

The null hypothesis H_O

The **null hypothesis H_O** is always an **equality statement** about a **population parameter.**
The null hypothesis H_O contains an equal sign.

I think the value of the population proportion is equal to .45 **H_O:** $\mu_x = 16$

I think the value of the population mean is equal to 16 **H_O:** $\mu_x = 16$

I think the value of the population standard deviation is equal to 2.37 **H_O:** $\sigma_x = 2.37$

The Alternate Hypothesis H_1

The **Alternate Hypothesis H_1** is an **inequality that disagrees with the null Hypothesis H_O**

There are only 3 ways that you can state an inequality that disagrees with a statement about equality

The alternate hypothesis **H_1** is always a statement that contains
one of the following **inequality signs.** $<$ or $>$ or $\neq$

Example 1

If **H_O** is I think the **population proportion is equal to .45** H_0: $p = .45$
Then H_1 could be any **one** of the following claims:

H_1: $p < .45$
I think the
population proportion
is less than .45

H_1: $p > .45$
I think the
population proportion
is more than .45

H_1: $p \neq .45$
I think the
population proportion
is not equal to .45

Example 2:

If **H_O** is I think the **population mean is equal** to 6.9
Then H_1 could be any **one** of the following claims:

H_1: $\mu_x < 6.9$
I think the **population**
mean is less than 6.9

H_1: $\mu_x > 6.9$
I think the **population**
mean is more than 6.9

H_1: $\mu_x \neq 6.9$
I think the **population**
mean is not equal to 6.9

If **H_O** is H_1: $\sigma_x = .45$
I think the **population standard deviation is equal to .45**
Then H_1 could be any **one** of the following claims:

H_1: $\sigma_x < .45$
I think the
population
standard deviation
is less than .45

H_1: $\sigma_x > .45$
I think the
population
standard deviation
is more than .45

H_1: $\sigma_x \neq .45$
I think the
population
standard deviation
is not equal to .45

The purpose of Hypothesis Testing is to test the null hypothesis

H_o: $\mu_x = 16$

and decide if you will reject it as false or not reject it.

We wish to test the null hypothesis that the population **mean is equal to** 16 **H_o:** $\mu = 16$

We cannot buy and weigh every single item the company makes to determine if the **population mean is equal to 16**. In some cases that is physically impossible, in others it is just too expensive

so

We Take a Random Sample of size n and
compare the hypothesis about the value of the population mean μ_x
to the value of the sample mean $\bar{x}$

We cannot expect the sample mean to be exactly 16 oz. even if the company's hypothesis that $\mu = 16$ has merit. A sample mean cannot accurately predict a population mean with 100% confidence.

To test the null hypothesis we always **"Take a Sample" of the Population**

We take a sample 50 boxes of candy. The **sample mean is 15.7 oz.** The customer is delighted. The sample mean is less than the value of the population mean stated by the company. **H_o:** $\mu = 16$ The customer declares he is correct and the company is wrong. The company says that you cannot expect a single sample mean to be exactly equal to the claim about the population mean. The company says if the sample mean is **"close enough"** to the declared value for the population mean than we should believe the company's claim that **H_o:** $\mu = 16$. The customer agrees that the sample mean can't be expected to match the population mean but the customer feels that the sample mean of 15.7 is just not "**close enough**" to 16 to believe that the population mean is equal to 16.

The question of who to believe must be based on what you mean by **"close enough"**

If the **sample mean of 15.7 is determined to be close enough** to the value claimed in the null **hypothesis of 16** then we **cannot reject the null hypothesis H_o** that the population mean is equal to 16 oz.

If the sample mean of 17.7 is determined to not be close enough to the value claimed in the null **hypothesis of 16** then **we cannot support the hypothesis statement that the population mean is equal to 24 oz**. We must accept the claim of the customer.

The purpose of Hypothesis testing is to decide if the difference between the sample mean and the value claimed in the null hypothesis H_o is large enough to enough to reject the null hypothesis (Reject H_o) and accept the claim made by H_1

or conclude that the difference is not that significant and state that we cannot reject the null hypothesis (Do not Reject H_o)

Stating the Conclusion to a Hypothesis Test

There are only two possible outcomes from a Hypotheses Test based on the $\mathbf{H_o}$ statement:

We Reject H_o or We Do Not Reject H_o.

Once the decision to **Reject H_o or Do Not Reject H_o** has been made a final conclusion must be stated. This statement is a **sentence written in English** that states the conclusion **based on the actual problem.**

Reject H_o

When we say we **Reject H_o** we **reject the statement of equality** and state that **there is sufficient evidence to accept the alternate hypothesis H_1** at the level of significance α that was used in the calculations. This means that the difference between the value of the parameter stated in $\mathbf{H_o}$ and the value from the sample **is too large** to accept that $\mathbf{H_o}$ is true. The **difference is significant** enough to reject $\mathbf{H_o}$. We must support the claim of inequality made in $\mathbf{H_1}$. The English statement we use to express this is given below. The **"inequality" phrase** will be the actual wording from the problem.

Reject H_o

There is sufficient evidence at the α level to **support the claim of "inequality" in H_1**

Do Not Reject H_o

When we say we **Do Not Reject H_o** we we are saying that the difference between value of the parameter stated in $\mathbf{H_o}$ and the sample **is not significant enough** to reject the equality statement $\mathbf{H_o}$. We **cannot reject H_o** (that they are equal) at the level of significance stated. We are saying the hull hypotheses **could be true**. We can **never accept** the Null Hypothesis.

Do Not Reject H_o :

There is **not sufficient** evidence at the α level to reject
the "statement of equality in $\mathbf{H_o}$ "

or

There is not sufficient evidence at the α level to support
the "statement of inequality in $\mathbf{H_1}$ "

Example 1
A claim about the Population Proportion p

Test the claim at a $\alpha = .10$ significance level that the proportion of FLC day time students that visit the library at least twice a month is **not equal to .65**

H_0 $p = .65$

the proportion of FLC day time students that visit the library at least twice a month **is equal to .65**

H_1 $p \neq .65$

the proportion of FLC day time students that visit the library at least twice a month **is not equal to .65**

Reject H_0

If our sample data leads us to **Reject H_0**: We conclude

There **is sufficient evidence** at the $\alpha = .10$ level **to support the claim** that the proportion of FLC day time students that visit the library at least twice a month **is not equal to .65**

Do Not Reject H_0

If our sample data leads us to **Do Not Reject H_0**: We conclude

There is **not sufficient evidence** at the $\alpha = .10$ level **to reject the hypothesis** that the proportion of FLC day time students that visit the library at least twice a month **is equal to .65**

or

There is **not sufficient evidence** at the $\alpha = .10$ level **to support the claim** that the proportion of FLC day time students that visit the library at least twice a month **is not equal to .65**

Example 2

A claim about the Population Mean μ

Use a $\alpha = .10$ significance level to test the student's claim that the average length of time a person owns a new car is **less than 5.6 years.**

H_O $\mu = 5.6$

the average length of time a person owns a new car **is equal to 5.6 years.**

H_1 $\mu < 5.6$

the average length of time a person owns a new car **is less than 5.6 years.**

Reject H_O

If our sample data leads us to **Reject H_O**: We conclude

There **is sufficient evidence** at the $\alpha = .10$ level **to support the claim** that the average length of time a person owns a new car is **less than 5.6 years.**

Do Not Reject H_O

If our sample data leads us to **Do Not Reject H_O**: We conclude

There is **not sufficient evidence** at the $\alpha = .10$ level to **reject the hypothesis** that the average length of time a person owns a new car is **equal to 5.6 years.**

or

There is **not sufficient evidence** at the $\alpha = .10$ level **to support the claim** that the average length of time a person owns a new car is **less than 5.6 years.**

Example 3

A claim about the Population Standard Deviation σ

Test the claim at a $\alpha = .01$ significance level that the **standard deviation** for the length of movies is **more than 10 minutes.**

H_O $\sigma = 10$

the standard deviation for
the length of movies
is **equal to 10 minutes**

H_1 $\sigma > 10$

the standard deviation for
the length of movies
is **more than 10 minute**

Reject H_O

If our sample data leads us to **Reject H_O**: We conclude

There **is sufficient evidence** at the $\alpha = .01$ level **to support the claim** that the standard deviation for the length of movies is **more than 10 minutes.**

Do Not Reject H_O

If our sample data leads us to **Do Not Reject H_O**: We conclude

There is **not sufficient evidence** at the $\alpha = .01$ level to **reject the hypothesis** that the standard deviation for the length of movies is equal to **10 minutes.**

or

There is **not sufficient evidence** at the $\alpha = .01$ level **to support the claim that** the standard deviation for the length of movies is **more than 10 minutes.**