## Chapter 7–4C: Examples of Constructing a Confidence Interval for the true value of the Population Standard Deviation $\sigma$ for a Normal Population.

### Example 1

Construct a 95% confidence interval for the true value of the **population standard deviation** $\sigma$ if the **population is normal**. **A random sample of size** 31($n = 31$) results in a **sample mean** ($\bar{x}$ ) of $\bar{x} = 12.8$ and a **sample standard deviation** of $s_x = 1.6$

The population is **given as normal so we can use the Chi–Square** $\chi^2$ **Table.**

The confidence level is 95% so $\alpha = .05$ If $\alpha = .05$ then $\alpha/2 = .025$ $\mathrm{DF} = n - 1 = 31 - 1 = 30$

$$\bar{x} = 12.8 \qquad s_x = 1.6 \qquad n = 31 \qquad \text{DF} = 30 \qquad \alpha/2 = .025$$

$$\chi^2_{\mathrm{L}} = 16.781 \qquad \chi^2_{\mathrm{R}} = 46.979$$

The Confidence Interval for the **population standard deviation** $\alpha$ is given by

$$\sqrt{\frac{(n-1)(s_x)^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)(s_x)^2}{\chi^2_L}}$$

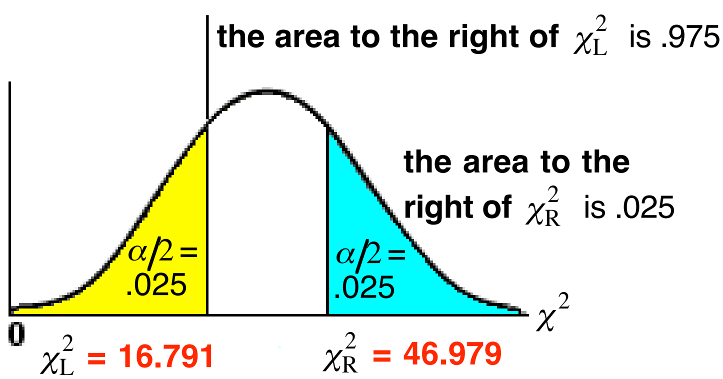
$$\sqrt{\frac{(30)(1.6)^2}{46.979}} < \sigma < \sqrt{\frac{(30)(1.6)^2}{16.791}}$$

$$1.28 < \sigma < 2.14$$

**Confidence Interval Statement:**

I am 95% confident that the interval $1.28 < \sigma < 2.14$ contains the value of the true population standard deviation $\alpha$

Finding $\chi^2_{\mathrm{L}}$ and $\chi^2_{\mathrm{R}}$



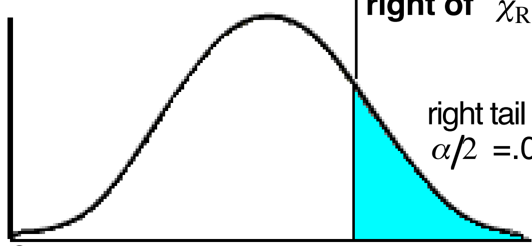
## Example 1 detailed explanation for finding $\chi^2_R$

To find the $\chi^2_R$ value for a **right tail area of** 0.025 **and Degrees of Freedom** 30

**use the part of the $\chi^2$ table shown below.**

**the $\chi^2_R$ value is** $+t_{\alpha/2}$ **= 46.979**

**DF = 30**




| Chi-Square | Distribution: | | Critical | Values | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Area | to the right of | | the Chi-Square value | | | | | |
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |

## Example 1 detailed explanation for finding $\chi^2_L$

**To find $\chi^2_L$ you must use the AREA TO THE RIGHT of $\chi^2_L$**

**if the left tail has an area of .025 then the area to the RIGHT of  is .975**

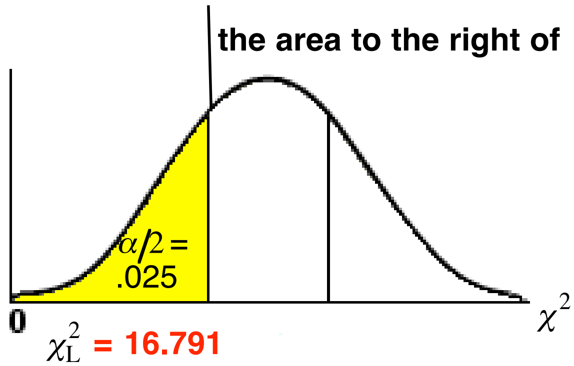
To find the $\chi^2_L$ value for a **right tail area of** **0.975** **and Degrees of Freedom** **30**

**use the part of the $\chi^2$ table shown below**

**the $\chi^2_L$ value is $+t_{\alpha/2}$ = 16.791**

Degrees of Freedom = 30

**the area to the right of $\chi^2_L$** is .975




| Chi-Square | Distribution: | Critical | Values | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Area | to the right of | the Chi-Square value | | | | | | | |
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |

## Example 2

Construct a 99 % confidence interval for the true value of the **population standard deviation** $\sigma$ if the **population is normal**. **A random sample of size** 71 ( $n = 71$ ) results in a **sample mean** ( $\bar{x}$ ) of $\bar{x} = 72.1$ and a **sample standard deviation** of $s_x = 5.28$

The population is **given as normal so we can use the Chi–Square** $\chi^2$ **Table.**

The confidence level is 99 % so $\alpha = .01$ and $\alpha/2 = .005$ $\quad$ $\text{DF} = n - 1 = 71 - 1 = 70$

$\bar{x} = 72.1 \quad s_x = 5.28 \quad n = 71 \quad$ DF = 70 $\quad \alpha/2 = .005$

$\chi^2_L = 43.275 \quad \chi^2_R = 104.215$

The Confidence Interval for the **population standard deviation** $\sigma$ is given by

$$\sqrt{\frac{(n-1)(s_x)^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)(s_x)^2}{\chi^2_L}}$$

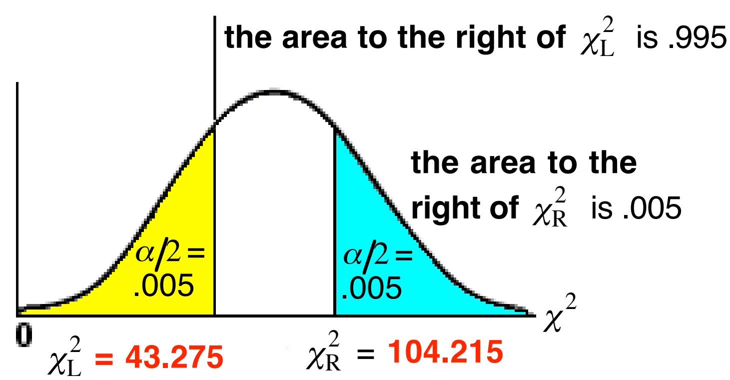
$$\sqrt{\frac{(70)(5.28)^2}{104.215}} < \sigma < \sqrt{\frac{(70)(5.28)^2}{43.275}}$$

$$4.33 < \sigma < 6.72$$

**Confidence Interval Statement:**

I am 99 % confident that the interval $4.33 < \sigma < 6.72$ contains the value of the true population standard deviation $\alpha$

Finding $\chi^2_L$ and $\chi^2_R$




| Chi–Square | Distribution: | Critical | Values | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Area to the RIGHT of the Chi-Square value | | | | | | | | |
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |

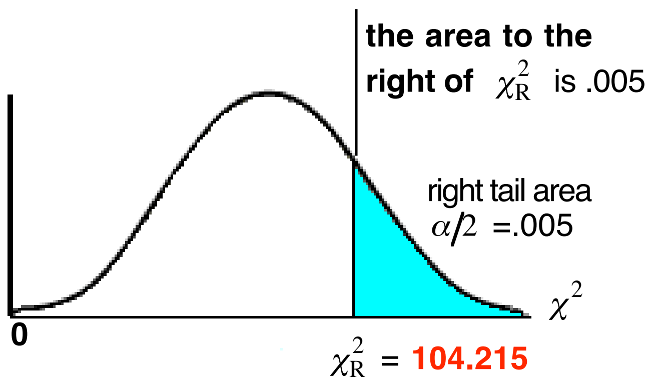
## Example 2 detailed explanation for finding $\chi^2_R$

**Finding $\chi^2_R$**

To find the $\chi^2_R$ value for a **right tail area of** **0.005** **and Degrees of Freedom** **70** **use the part of the $\chi^2$ table shown below.**

**the $\chi^2_R$ value is** $+t_{\alpha/2}$ **= 104.215**

**DF = 70**




| Chi-Square Distribution: Critical Values | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Area to the RIGHT of the Chi-Square value | | | | | | | | |
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |

**Example 2 detailed explanation for finding $\chi_L^2$**

**To find $\chi_L^2$ you must use the AREA TO THE RIGHT of $\chi_L^2$**

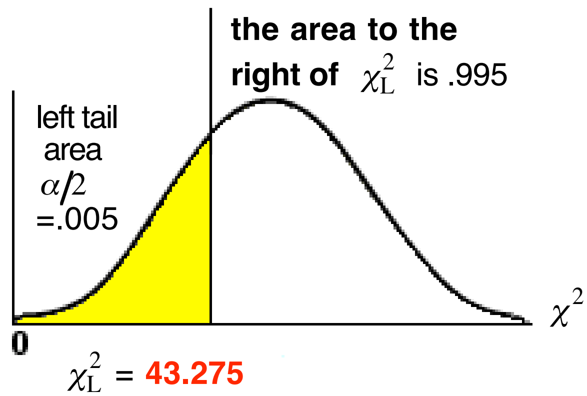
**if the left tail has an area of .005 then the area to the RIGHT of is .995**

To find the $\chi_L^2$ value for a **right tail area of** **0.995** **and Degrees of Freedom** **70**

**use the part of the $\chi^2$ table shown below**

**the $\chi_L^2$ value is $+t_{\alpha/2}$ = 43.275**

**DF = 70**




| Chi–Square | Distribution: | Critical | Values | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Area to the right of the Chi-Square value | | | | | | | | | 
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |

## Example 3

The California Consumer Protection Agency decides to conduct an investigation into the quality of car tires. They sample 41 randomly selected tires and find an average tread wear of 79,000 and a standard deviation of 2,300 miles. Construct a 90% confidence interval for the true value of the **population standard deviation** $\sigma$ . Assume that if the population of car tire tread wear is normally distributed.

The population is **given as normal so we can use the Chi–Square** $\chi^2$ **Table.**

The confidence level is 90% so $\alpha = .10$ and $\alpha/2 = .05$ $\quad$ $\text{DF} = n-1 = 41-1 = 40$

$$\bar{x} = 79{,}000 \qquad s_x = 2{,}300 \qquad n = 41 \qquad \text{DF} = 40 \qquad \alpha/2 = .05$$

$$\chi^2_{\text{L}} = 26.509 \qquad \chi^2_{\text{R}} = 55.758$$

The Confidence Interval for the **population standard deviation** $\alpha$ is given by

$$\sqrt{\frac{(n-1)(s_x)^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)(s_x)^2}{\chi^2_L}}$$

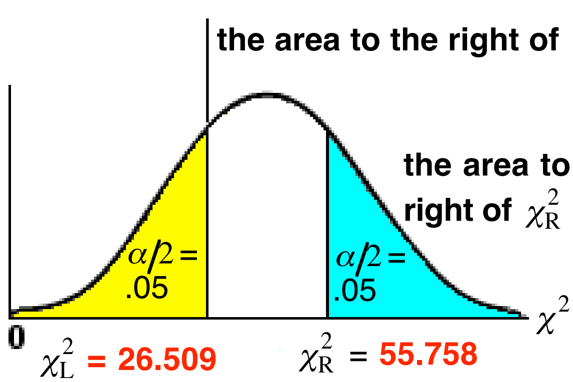
$$\sqrt{\frac{(40)(2300)^2}{55.758}} < \sigma < \sqrt{\frac{(40)(2300)^2}{26.509}}$$

$$1948.1 < \sigma < 2825.3$$

**Confidence Interval Statement:**

I am 99% confident that the interval $1948.1 < \sigma < 2825.3$ contains the value of the true population standard deviation $\alpha$

Finding $\chi^2_{\text{L}}$ and $\chi^2_{\text{R}}$




| Chi–Square Distribution: Critical Values | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Area to the RIGHT of the Chi-Square value | | | | | | | | | |
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 40 | 20.707 | 21.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |

## Example 3 detailed explanation for finding $\chi^2_R$

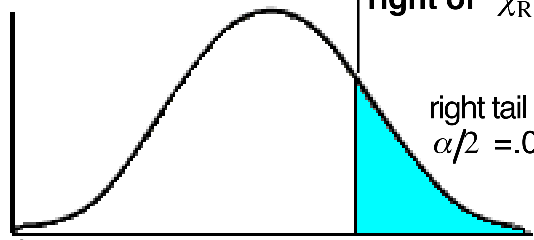
**Finding $\chi^2_R$**

To find the $\chi^2_R$ value for a **right tail area of** $\alpha/2$ **= 0.05 and Degrees of Freedom 40**

**use the part of the $\chi^2$ table shown below.**

**the $\chi^2_R$ value is** $+t_{\alpha/2}$ **= 55.758**

**DF = 40**




| Chi-Square Distribution: Critical Values | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Area to the RIGHT of the Chi-Square value | | | | | | | | | |
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 40 | 20.707 | 21.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |

## Example 3 detailed explanation for finding $\chi^2_L$

To **find** $\chi^2_L$ **you must use the AREA TO THE RIGHT of** $\chi^2_L$

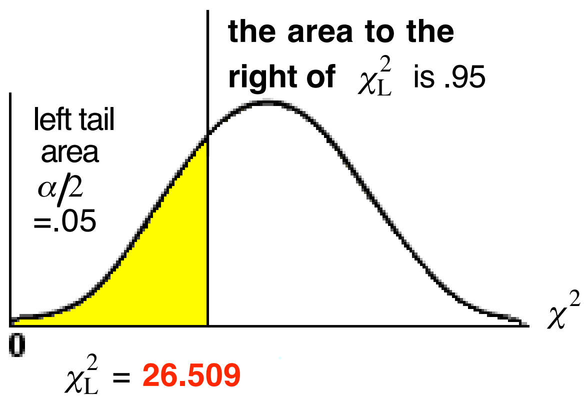
**if the left tail has an area of .05 then the area to the RIGHT of  is .95**

To find the $\chi^2_L$ value for a **right tail area of** 0.95 **and Degrees of Freedom** 40

**use the part of the $\chi^2$ table shown below.**

**the $\chi^2_L$ value is $+t_{\alpha/2}$ = 26.509**

**DF = 40**




| Chi-Square | Distribution: | Critical | Values | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Area | to the right of | the Chi-Square value | | | | | | | |
| D of F | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 40 | 20.707 | 21.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |