

## Finding Two Tail Critical Values for a Chi-Square $\chi^2$ Distribution

A Confidence Interval is based on the Confidence Level we require  
The **Confidence Level** is  $1 - \alpha$  and represents the area between the Left and Right Tail Areas.  
If the area between the two tails is  $1 - \alpha$  then the total area in both tails is  $\alpha$ .

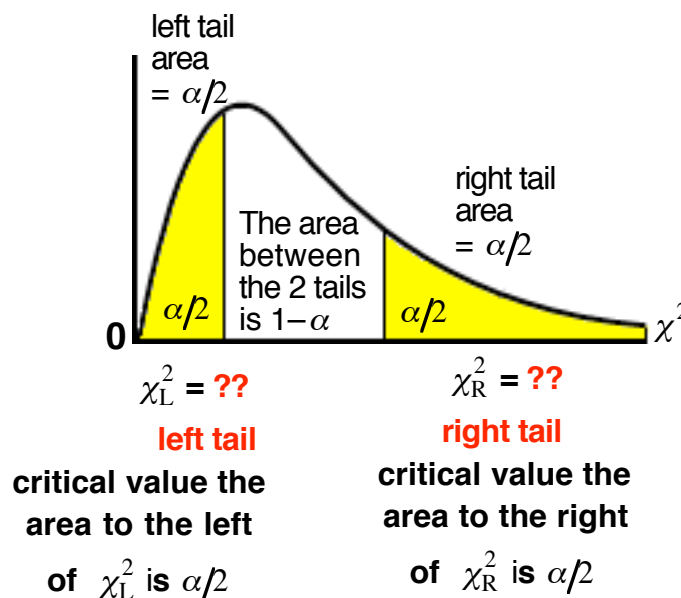
### Two Tail Critical Values

If the total area in both tails is  $\alpha$   
and this area is **divided equally** between the the left and right tails  
then the left and right tails will each have an area of  $\alpha/2$

A **Critical Value** for a Chi-Square  $\chi^2$  Distribution is a  $\chi^2$  **value on the  $\chi^2$  axis** that is the vertical boundary **separating the area in one tail of the graph from the remaining area**.

The vertical boundary **separating the area in the Left Tail** of the graph from the remaining area is the **left tail critical value** or  $\chi_L^2$

The vertical boundary **separating the area in the Right Tail** of the graph from the remaining area is the **right tail critical value** or  $\chi_R^2$

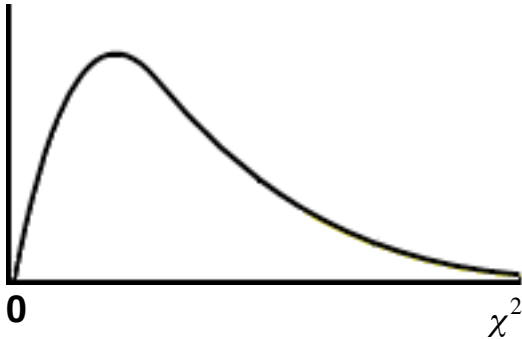


**Note:** In a two tail graph the left tail area and the right tail area both have a value of  $\alpha/2$ . The two tail areas do not look equal because the distribution is not normal. It is skewed left causing the tails with equal area to look different. The left tail area is tall and narrow shaped and the right tail area is short and wide. In fact the right tail continues indefinitely without end.

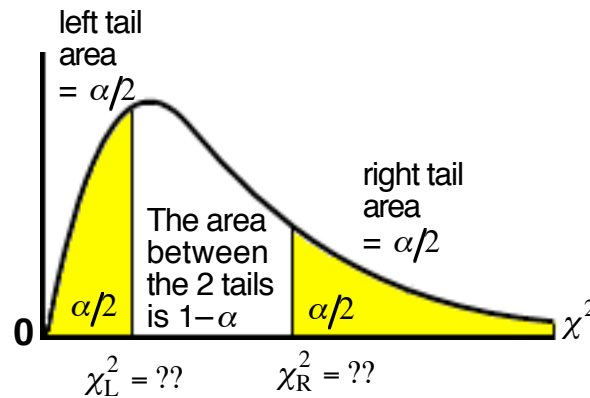
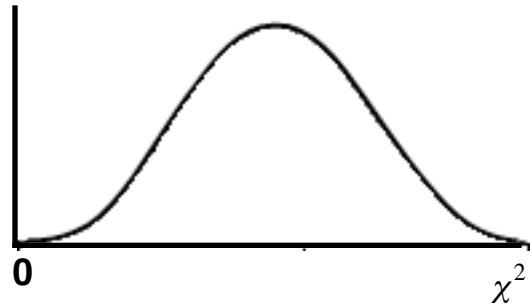
The  $\chi^2$  distribution is **NOT Normal**. It is skewed to the Right

As the the number of degrees of freedom increases (DF = n-1) increases  
the  $\chi^2$  distribution becomes **more normal**.

DF = 10



DF = 50



The  $\chi^2$  starts at 0 and increases in the positive direction.

No  $\chi^2$  values are negative.

The left tail critical values will be less than the right tail critical value.

The left tail critical value will be a different positive number than the right tail critical value.

The  $\chi^2$  table below is used to find the  $\chi_L^2$  value for the **LEFT TAIL Critical Value**

and the  $\chi_R^2$  value for the **RIGHT TAIL Critical Value**

**the right tail critical value  $\chi_R^2$  is based on the area in the RIGHT TAIL**

**and the the degrees of freedom equal to n – 1**

**The left tail critical value  $\chi_L^2$  is based on the area to the RIGHT of the left tail area**

**and the the degrees of freedom equal to n – 1**

Chi-Square Distribution: Critical Values										
Deg. of Freedom	Area in the Right Tail (One Tail)									
	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
This is only a portion of the entire Chi-Square Table										

### Example 1

Finding  $\chi_L^2$  and  $\chi_R^2$  if the total tail area  $\alpha = .05$  with  $n = 31$

If  $\alpha = .05$  then  $\alpha/2 = .025$  and  $DF = 31 - 1 = 30$

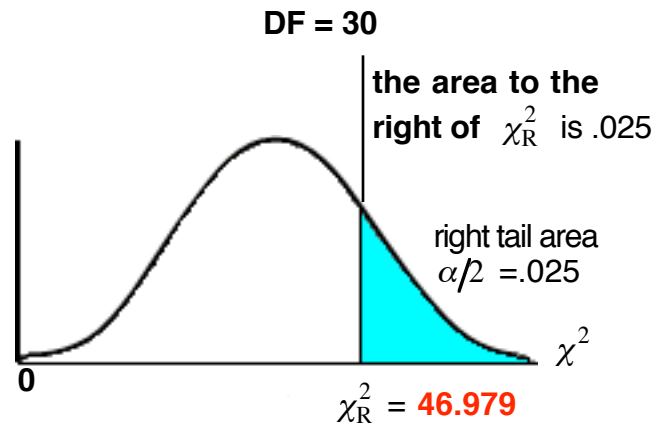
#### Example 1 detailed explanation for finding $\chi_R^2$

To find the **right tail critical value**  $\chi_R^2$

for a right tail area of  $\alpha/2 = 0.025$  and the Degrees of Freedom 30

use the part of the t table shown below.

the  $\chi_R^2$  value is = 46.979



Chi-Square Distribution: Critical Values										
	Area to the right of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672

**Example 1 detailed explanation for finding  $\chi^2_L$**

To find the  $\chi^2_L$  value for a **left tail area of  $\alpha/2 = .025$**  and **Degrees of Freedom 30**  
 you must use **the AREA TO THE RIGHT of  $\chi^2_L$**

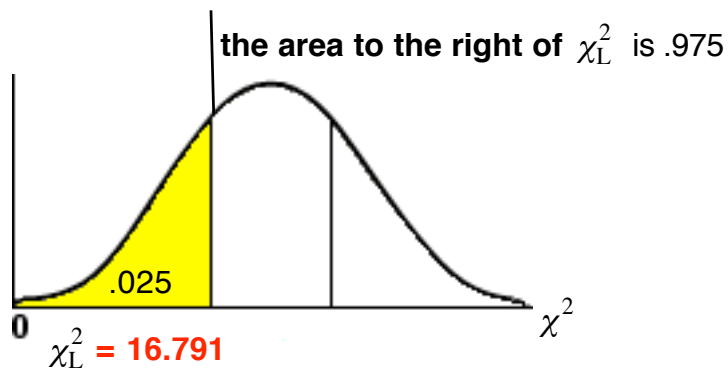
**if the left tail has an area of .025 then the area to the RIGHT of is .975**

To find the **left tail critical value  $\chi^2_L$**

for a right tail area of **0.975** and the Degrees of Freedom **30**  
 use the part of the t table shown below.

the  $\chi^2_L$  value is = **16.791**

Degrees of Freedom = 30



Chi-Square Distribution: Critical Values										
	Area to the right of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672

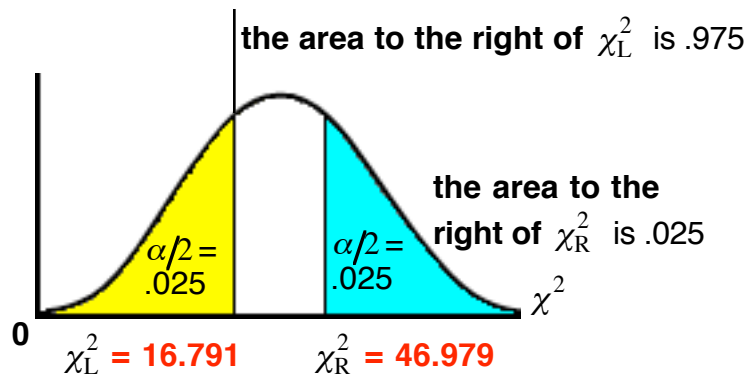
Example 1

Finding  $\chi_R^2$  and  $\chi_L^2$  at the same time

The row of the  $\chi^2$  table with the DF = **30** contains both the  $\chi_L^2$  value for the **LEFT TAIL Critical Value** and the  $\chi_R^2$  value for the **RIGHT TAIL Critical Value** for DF = **30**

The  $\chi_R^2$  value for the **RIGHT TAIL Critical Value** is read with a **right tail area** of **0.025**

The  $\chi_L^2$  value for the **Left TAIL Critical Value** is read with **an area to the right of the left tail area** of .025 which is  $1 - .025 =$  **0.975**



Chi-Square	Distribution: Critical Values									
	Area to the right of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
<b>30</b>	13.787	14.953	<b>16.791</b>	18.493	20.599	40.256	43.773	<b>46.979</b>	50.892	53.672

the  $\chi_L^2$  value is **16.791**

the  $\chi_R^2$  value is **46.979**

### Example 2

Finding  $\chi_L^2$  and  $\chi_R^2$  if the total tail area  $\alpha = .01$  with  $n = 71$

If  $\alpha = .01$  then  $\alpha/2 = .005$  and  $DF = 71 - 1 = 70$

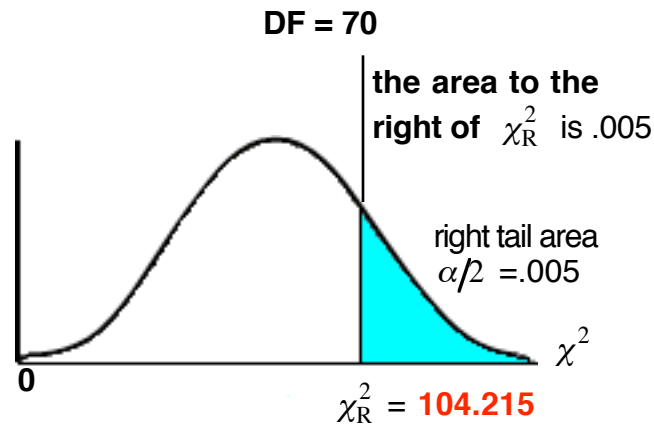
### Example 1 detailed explanation for finding $\chi_R^2$

To find the **right tail critical value  $\chi_R^2$**

for a right tail area of  $\alpha/2 =$  0.005 and the Degrees of Freedom 70

use the part of the t table shown below.

the  $\chi_R^2$  value is = **104.215**



Chi-Square Distribution: Critical Values										
	Area to the RIGHT of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
<b>70</b>	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	<b>104.215</b>

**Example 2 detailed explanation for finding  $\chi_L^2$**

To find the  $\chi_L^2$  value for a **left tail area of  $\alpha/2 = .005$**  and **Degrees of Freedom 70**  
 you must use **the AREA TO THE RIGHT of  $\chi_L^2$**

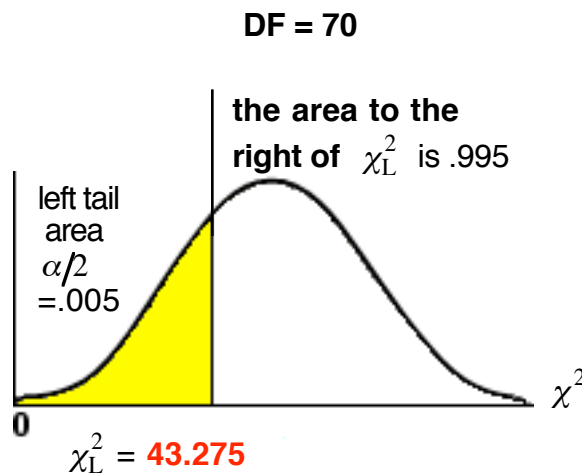
**if the left tail has an area of .005 then the area to the RIGHT of is .995**

To find the **left tail critical value  $\chi_L^2$**

for a right tail area of **0.995** and the Degrees of Freedom **70**

use the part of the t table shown below.

the  $\chi_L^2$  value is = **43.275**



Chi-Square Distribution: Critical Values										
	Area to the right of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215

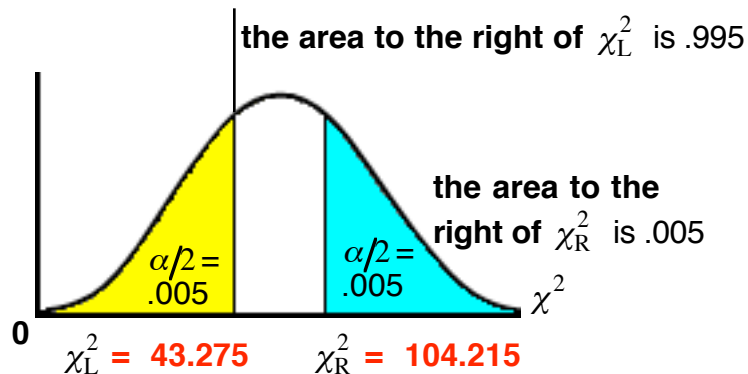
### Example 2

Finding  $\chi_R^2$  and  $\chi_L^2$  at the same time

The row of the  $\chi^2$  table with the DF = **70** contains both the  $\chi_L^2$  value for the **LEFT TAIL Critical Value** and the  $\chi_R^2$  value for the **RIGHT TAIL Critical Value** for DF = **70**

The  $\chi_R^2$  value for the **RIGHT TAIL Critical Value** is read with a **right tail area** of **0.005**

The  $\chi_L^2$  value for the **Left TAIL Critical Value** is read with **an area to the right of the left tail area** of .005 which is  $1 - .005 =$  **0.995**



Chi-Square	Distribution: Critical Values									
	Area to the RIGHT of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215

the  $\chi_L^2$  value is = **43.275**

the  $\chi_R^2$  value is = **104.215**

### Example 3

Finding  $\chi_L^2$  and  $\chi_R^2$  if the total tail area  $\alpha = .10$  with  $n = 41$

If  $\alpha = .10$  then  $\alpha/2 = .05$  and  $DF = 41 - 1 = 40$

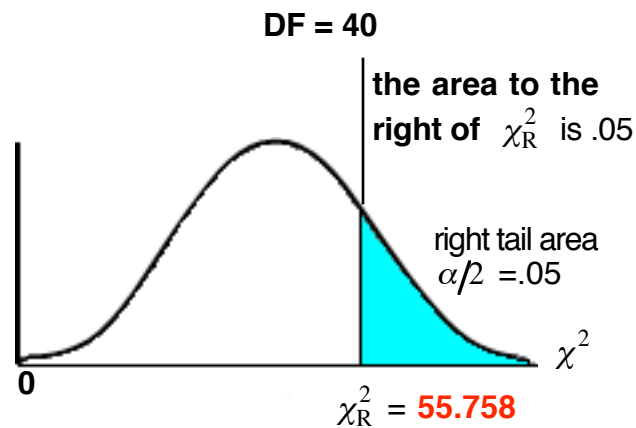
**Example 1 detailed explanation for finding  $\chi_R^2$**

To find the **right tail critical value  $\chi_R^2$**

for a right tail area of  $\alpha/2 =$  0.05 and the Degrees of Freedom 40

use the part of the t table shown below.

the  $\chi_R^2$  value is = **55.758**



Chi-Square Distribution: Critical Values										
	Area to the RIGHT of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
<b>40</b>	20.707	21.164	24.433	26.509	29.051	51.805	<b>55.758</b>	59.342	63.691	66.766

**Example 3 detailed explanation for finding  $\chi_L^2$**

To find the  $\chi_L^2$  value for a left tail area of  $\alpha/2 = .05$  and Degrees of Freedom 40 you must use the AREA TO THE RIGHT of  $\chi_L^2$

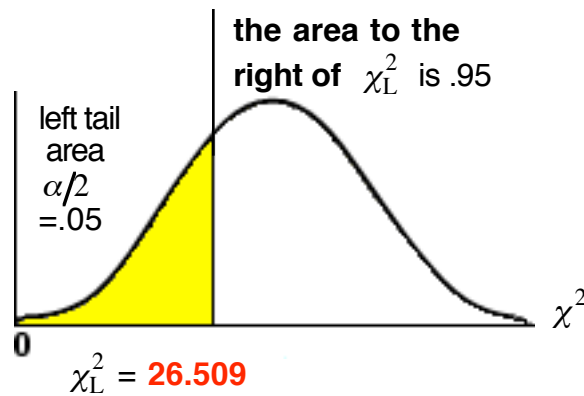
if the left tail has an area of .05 then the area to the RIGHT of is .95

To find the left tail critical value  $\chi_L^2$

for a right tail area of 0.95 and the Degrees of Freedom 40

use the part of the t table shown below.

the  $\chi_L^2$  value is = 26.509



Chi-Square Distribution: Critical Values										
	Area to the right of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
40	20.707	21.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766

### Example 3

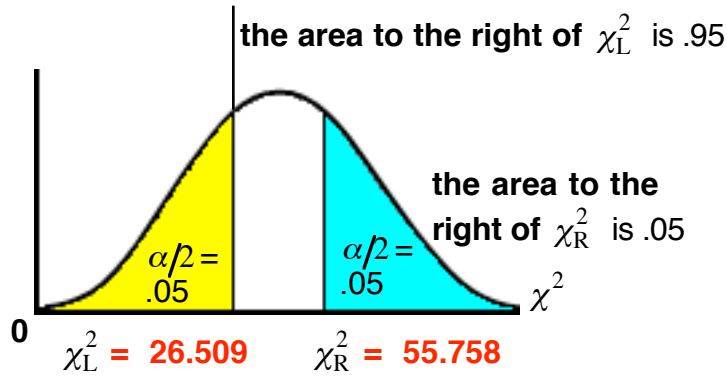
Finding  $\chi_R^2$  and  $\chi_L^2$  at the same time

The row of the  $\chi^2$  table with the DF = **40** contains both the  $\chi_L^2$  value for the **LEFT TAIL Critical Value** and the  $\chi_R^2$  value for the **RIGHT TAIL Critical Value**

for DF = **40**

The  $\chi_R^2$  value for the **RIGHT TAIL Critical Value** is read with a **right tail area** of **0.05**

The  $\chi_L^2$  value for the **Left TAIL Critical Value** is read with an **area to the right of the left tail area** of .05 which is  $1 - .05 =$  **0.95**



Chi-Square	Distribution: Critical Values									
	Area to the RIGHT of the Chi-Square value									
D of F	0.995	0.99	0.975	0.95	0.9	0.10	0.05	0.025	0.01	0.005
<b>40</b>	20.707	21.164	24.433	<b>26.509</b>	29.051	51.805	<b>55.758</b>	59.342	63.691	66.766

the  $\chi_L^2$  value is **26.509**

the  $\chi_R^2$  value is **55.758**