Chapter 7–3C:

Examples of constructing a Confidence Interval for the true value of the population mean μ with σ NOT known but s_x IS known.

Example 1

Construct a 95% confidence interval for the true value of the population mean μ if the **population is normal**, the **sample mean** is $\bar{x} = 17.35$, the **random sample size** is $n = 20$ and the **sample standard deviation** s_x is known to be $s_x = 1.35$

The population is **given as normal but we do not know σ.**
We do know s_x so we must use the t table.

The confidence level is 95% so $\alpha = .05$ If $\alpha = .05$ then $\alpha/2 = .025$ $\text{DF} = n - 1 = 20 - 1 = 19$

$\bar{x} = 17.35$ $s_x = 1.35$ $n = 20$ $\alpha/2 = .025$ DF = 19

$$E = +t_{\alpha/2} \bullet \frac{s_x}{\sqrt{n}} = 2.093 \bullet \frac{1.35}{\sqrt{20}} = .63$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$17.35 - .63 < \mu < 17.35 + .63$$

$$16.72 < \mu < 17.98$$

Confidence Interval Statement:

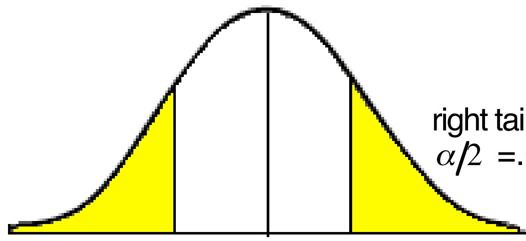
I am 95% confident that the interval $16.72 < \mu < 17.98$ contains the value of the true population mean μ.

Example 1
explanation for finding $+t_{\alpha/2}$

95% Confidence Level

$\alpha = .05$ so $\alpha/2 = .025$

N = 20 so DF = 20 − 1 = 19



To find the **positive t value**

for a right tail area of $\alpha/2$ = 0.025 and the Degrees of Freedom 19

use the part of the t table shown below

the positive t value is $+t_{\alpha/2}$ = 2.093

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
19	1.328	1.729	2.093	2.539	2.861

Example 2

Construct a 99% confidence interval for the true value of the population mean μ if the **population is skewed (not normal)**, the **sample mean** is $\bar{x} = 56.52$, the **random sample size** is $n = 41$ and the **sample standard deviation** s_x is known to be $s_x = 5.67$

The population is **given as skewed (not normal) but n > 30 so the sample can be assumed to be approximately normal. We know** s_x **(not** σ **) so we must use the t table.**

The confidence level is 99% so $\alpha = .01$ If $\alpha = .01$ then $\alpha/2 = .005$ $\text{DF} = n-1 = 41-1 = 40$

$$\bar{x} = 56.52 \quad s_x = 5.67 \quad \alpha/2 = .005 \quad n = 41 \quad \text{DF} = 40$$

$$E = +t_{\alpha/2} \bullet \frac{s_x}{\sqrt{n}} = 2.704 \bullet \frac{5.67}{\sqrt{41}} = 2.39$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$56.52 - 2.39 < \mu < 56.52 + 2.39$$

$$54.13 < \mu < 58.91$$

Confidence Interval Statement:

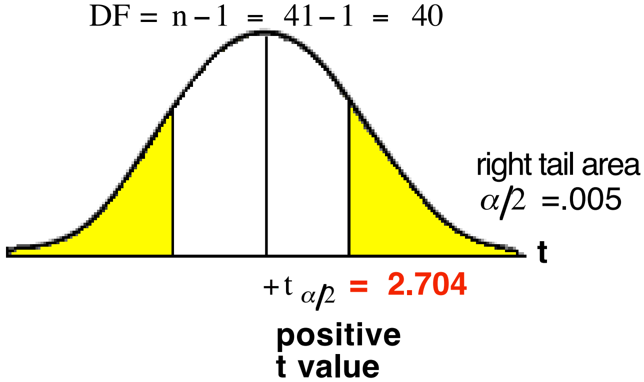
I am 99% confident that the interval $54.13 < \mu < 58.91$ contains the value of the true population mean μ .

Finding $+t_{\alpha/2}$

$\alpha = .01 \quad \alpha/2 = .005$

99% Confidence Level

$\text{DF} = n-1 = 41-1 = 40$



Example 2 detailed explanation for finding $+t_{\alpha/2}$

Finding the positive t score

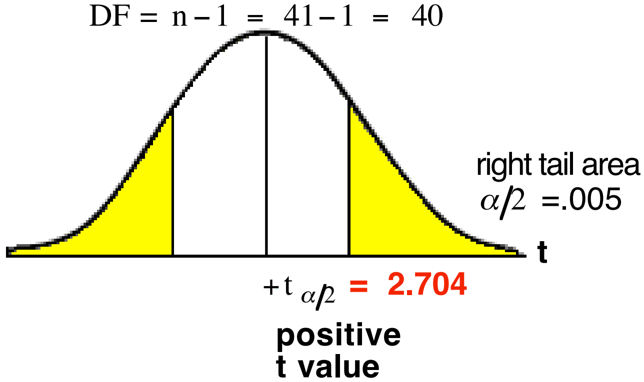
for an area of $\alpha = .01$ that is equally divided between the left and the right tail

$\alpha/2 = .005$ in the left tail and $\alpha/2 = .005$ in the right tail.

$\alpha = .01 \qquad \alpha/2 = .005$

99% Confidence Level

DF = n − 1 = 41 − 1 = 40



To find the **positive t value**

for a right tail area of $\alpha/2$ = 0.005 and the Degrees of Freedom 40

use the part of the t table shown below.

the positive t value is $+t_{\alpha/2}$ = 2.704

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
40	1.303	1.684	2.021	2.423	2.704

Example 3

A random sample of 71 FLC students shows that the average number of units completed in the Foell 2008 semester was 12.6 with a sample standard deviation 2.1 Construct a 98% confidence interval for the true population mean for the average number of units completed in the Foell 2008 semester by FLC students.

The shape of the population distribution **is not known but n > 30 so the sample can be assumed to be approximately normal. We know s_x (not σ) so we must use the t table.**

The confidence level is 98% so $\alpha = .02$ If $\alpha = .02$ then $\alpha/2 = .01$ $\text{DF} = n - 1 = 72 - 1 = 70$

$$\bar{x} = 12.6 \qquad s_x = 2.1 \qquad n = 71 \qquad \alpha/2 = .01 \qquad \text{DF} = 70$$

$$E \; = \; + t_{\alpha/2} \bullet \frac{s_x}{\sqrt{n}} \; = \; 2.381 \bullet \frac{2.1}{\sqrt{71}} \; = \; .59$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$12.60 - .59 \; < \; \mu \; < \; 12.60 + .59$$

$$12.01 \; < \; \mu \; < \; 13.19$$

Confidence Interval Statement:

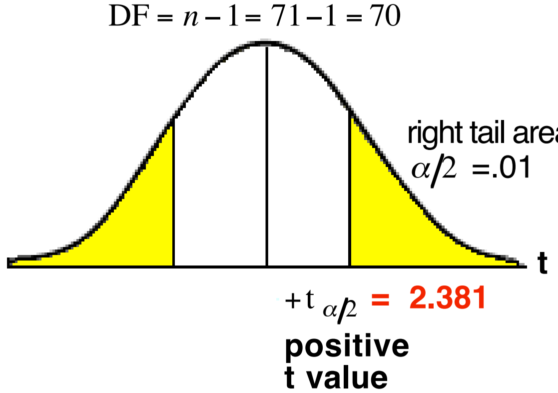
I am 98% confident that the interval $12.01 < \mu < 13.19$ contains the value of the true population mean μ.

Finding $+t_{\alpha/2}$

$\alpha = .02 \qquad \alpha/2 = .01$

98% Confidence Level

$\text{DF} = n - 1 = 71 - 1 = 70$



Example 3 detailed explanation for finding $+t_{\alpha/2}$

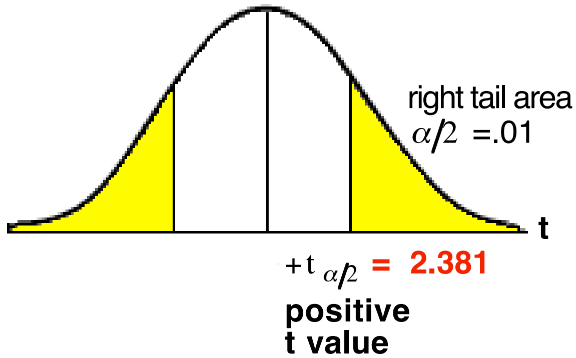
Finding the positive t score

for an area of $\alpha = .02$ that is equally divided between the left and the right tail $\alpha/2 = .01$ in the left tail and $\alpha/2 = .01$ in the right tail.

$\alpha = .02 \quad \alpha/2 = .01$

98% Confidence Level

$\text{DF} = n - 1 = 71 - 1 = 70$



To find the **positive t value**

for a right tail area of $\alpha/2$ = 0.01 and the Degrees of Freedom 70

use the part of the t table shown below.

the positive t value is $+t_{\alpha/2}$ = 2.381

t Distribution: Critical t Values					
Degrees of	Area In One Tail (Right Tail)				
Freedom	0.100	0.050	0.025	0.010	0.005
70	1.294	1.667	1.994	2.381	2.648

