Chapter 7–3A: Estimating A Population Mean μ with σ NOT known.

Creating a Confidence Interval for the True Value of the Population Mean μ
for a Normal Population (or n $>$ 30)
from a simple random sample of size n with a Confidence Level of $1-\alpha$
the population standard deviation is not known
but the sample standard deviation s_x is known

Requirements for using the Student t Distribution (t table)
to create a Confidence Interval for the Population Mean μ

1. The population standard deviation σ is not known but the sample standard deviation s_x is known.
2. The population is known to be normal **OR** the sample size is greater than 30 $(n > 30)$
3. The sample mean $\bar{x}$ and sample standard deviation s_x has been computed based on data from a simple random sample.

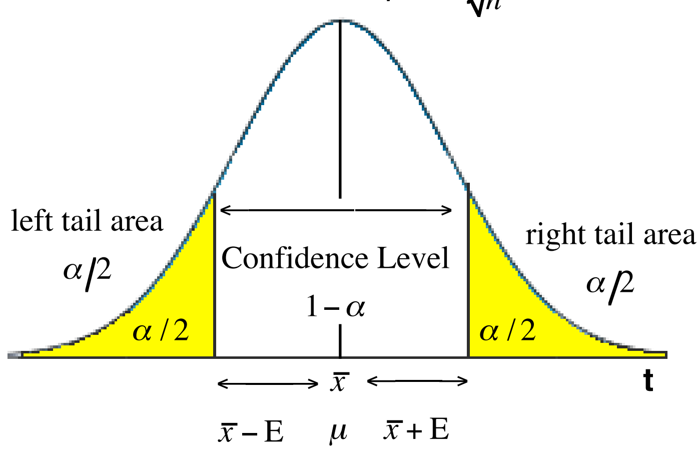
The **Confidence Interval** for the **population mean** μ

with σ NOT known but the sample standard deviation s_x is known

with a Confidence Level of $1-\alpha$ is given by

$$\bar{x} - E < \mu < \bar{x} + E$$

where $E = t_{\alpha/2} \bullet \dfrac{s_x}{\sqrt{n}}$



I am $1-\alpha$ confident that the actual value for the Population Mean μ will be between

$$\bar{x} - E < \mu < \bar{x} + E$$

I am $\alpha/2$ confident that the actual value for the Population Mean μ will be greater than

$$\bar{x} + E$$

I am $\alpha/2$ confident that the actual value for the Population Mean μ will be less than

$$\bar{x} - E$$

Requirements for using the Student t Distribution (t table)

If the population is normal or n > 30 and the population standard deviation σ is known then

$$E = z_{\alpha/2} \bullet \frac{\sigma_x}{\sqrt{n}}$$

1. If we do not know the value of the population standard deviation σ then we we cannot use the z table. In most cases the population standard deviation σ is **not known** but simple random sample of size n has been taken and the **sample mean and standard deviation** s_x **have been calculated**. Both of these sample values will be used to calculate the maximum error E and then create your confidence interval.

   When we do not know the value of σ we can use the sample standard deviation s_x in its place but we must make an adjustment. The value of the sample standard deviation s_x is not equal to the population standard deviation σ, especially in small samples. If we want to have the same confidence level for the maximum error E then we must widen the confidence interval by creating a larger value of E. The use of the normal Student t distribution in place of the standard normal Z distribution accomplishes this. The values for $t_{\alpha/2}$ are larger than the value for $z_{\alpha/2}$ for every value of α. The use of the larger $t_{\alpha/2}$ value instead of the $z_{\alpha/2}$ value creates larger value of E and a wider confidence interval for the same confidence level.

   If the population is normal or n > 30 and the population standard deviation σ is not known but the **standard deviation** s_x **is known** then

$$E = t_{\alpha/2} \bullet \frac{s_x}{\sqrt{n}}$$

2. If we do not know σ then we we cannot use the z table. If the population is **known to be normal** or the sample size n is greater than 30 ($n > 30$) then we can use the Student t Distribution but we must use s_x in place of the value of σ which is not known. We may not know if the population is normal or the sample size n may not be greater than 30 ($n > 30$). If this is the case then you cannot use the methods of this section to compute the maximum error E. You should increase the sample size so that $n > 30$ and then use the t table.

3. The single value for the sample mean $\bar{x}$ is the single best "guess" about what the true population mean μ really is. We call $\bar{x}$ the best **point estimate** of the true population mean μ. The sample mean is an unbiased estimator of the population mean μ. The distribution of the sample means centers about the value of the population mean. The distribution of the sample means tends to have less variation than the distribution of the population proportion and the population standard deviation.

