Chapter 7–2B:

Examples of constructing a Confidence Interval for the true value of the population mean μ with σ known.

Example 1

Construct a 95% confidence interval for the population mean μ if the **population is normal**, the **sample mean** is $\bar{x} = 5.2$, the **random sample size** is $n = 20$ and the **population standard deviation** σ is known to be $\sigma = .45$

The population is **given as normal so we can use the z table.**

The confidence level is 95% so $\alpha = .05$ If $\alpha = .05$ then $\alpha/2 = .025$

$$\bar{x} = 5.2 \qquad \sigma = .45 \qquad n = 20 \qquad \alpha/2 = .025$$

$$E = +z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}} = 1.96 \bullet \frac{.45}{\sqrt{20}} = .20$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$5.2 - .20 < \mu < 5.2 + .2$$

$$5.0 < \mu < 5.4$$

Confidence Interval Statement:

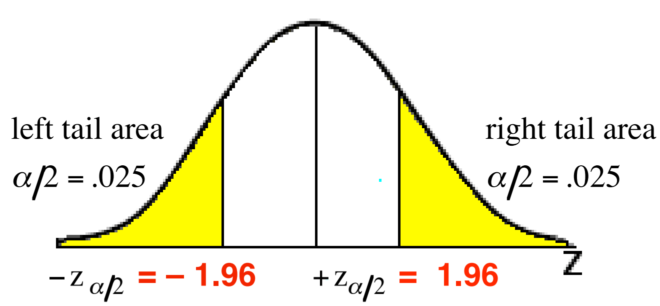
I am 95% confident that the interval $5.0 < \mu < 5.4$ contains the value of the true population mean μ.

Finding $+z_{\alpha/2}$

$$\alpha = .05$$

$$\text{Confidence Level} = 1 - \alpha$$

$$95\% \quad \text{Confidence Level}$$

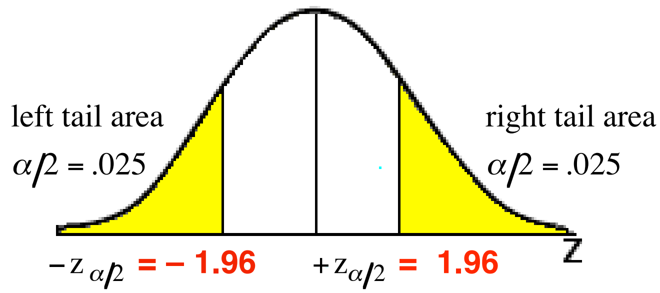


Example 1 detailed explanation for finding $+z_{\alpha/2}$

Finding the positive Z score

for an area of $\alpha = .05$ that is equally divided between the left and the right tail

$\alpha/2 = .025$ in the left tail and $\alpha/2 = .025$ in the right tail.



Find an area in the **body of the table** that is as close to .025 as possible

0.0250 **is an exact table value**

the negative Z score that has an .025 area to the left – 1.96

Z = – 1.96

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233

The positive $+z_{\alpha/2}$ = $|-z_{\alpha/2}|$

The positive $+z_{\alpha/2}$ value is Z = |– 1.96| = 1.96

we can verify this by looking for a positive z score with an .975 area to the left of $+z_{\alpha/2}$

Positive Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Example 2

Construct a 98% confidence interval for the population mean μ if the **population is skewed (not normal)**, the **sample mean** is $\bar{x} = 32.7$, the random **sample size** is $n = 40$ and the **population standard deviation** σ is known to be $\sigma = 3.75$

The population is given as skewed (NOT normal) but n > 30 so we can use the z table.

The confidence level is 98% so $\alpha = .02$ If $\alpha = .02$ then $\alpha/2 = .01$

$$\bar{x} = 32.7 \qquad \sigma = 3.75 \qquad n = 40 \qquad \alpha/2 = .01$$

$$E = +z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}} = 2.33 \bullet \frac{3.75}{\sqrt{40}} = 1.38$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$32.7 - 1.38 < \mu < 32.7 + 1.38$$

$$31.32 < \mu < 34.08$$

Confidence Interval Statement:

I am 98% confident that the interval $31.32 < \mu < 34.08$ contains the value of the true population mean μ
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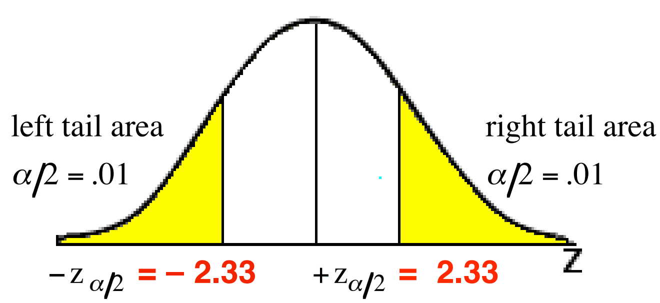
Finding $+z_{\alpha/2}$

$\alpha = .02$

Confidence Level $= 1 - \alpha$

98% Confidence Level



Example 2 detailed explanation for finding $+z_{\alpha/2}$

Finding the positive Z score

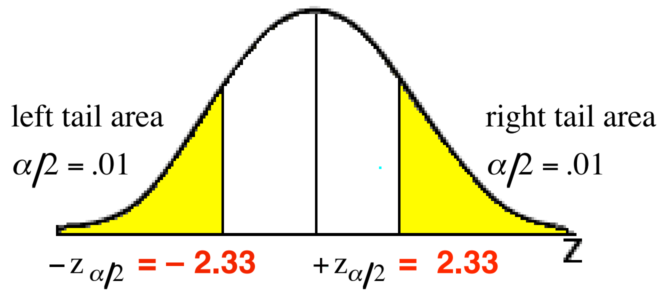
for an area of $\alpha = .02$ that is equally divided between the left and the right tail

$\alpha/2 = .01$ in the left tail and $\alpha/2 = .01$ in the right tail.

$\alpha = .02$

Confidence Level $= 1 - \alpha$

98% Confidence Level



Find an area in the **body of the table** that is as close to .01 as possible

0.0099 **is as close to .01 as possible**

the negative Z score that has an .01 area to the left is – 2.33

Z = – 2.33

Negative Z Scores									
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
–2.3	0.0107	0.0104	0.0102	**0.0099**	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084

The positive $+z_{\alpha/2}$ **=** $|-z_{\alpha/2}|$

The positive $+z_{\alpha/2}$ **value is Z = |– 2.33| = 2.33**

we can verify this by looking for a positive z score with an .99 area to the left of $+z_{\alpha/2}$

Positive Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
2.3	0.9893	0.9896	0.9898	**0.9901**	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916

Example 3

A random sample of 90 FLC students shows a mean age of 24.6 years. The standard deviation of the population of FLC students age is known to be 1.63 years. Construct a 99% confidence interval for the true population mean for the age of all FLC students.

The population is not given as normal but n > 30 so we can use the z table.

The confidence level is 99% so $\alpha = .01$ If $\alpha = .01$ then $\alpha/2 = .005$

$$\bar{x} = 24.6 \qquad \sigma = 1.63 \qquad n = 90 \qquad \alpha/2 = .005$$

$$E = +z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}} = 2.575 \bullet \frac{1.63}{\sqrt{90}} = .44$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$24.6 - .44 < \mu < 24.6 + .44$$

$$24.16 < \mu < 25.04$$

Confidence Interval Statement:

I am 99% confident that the interval $24.16 < \mu < 25.04$ contains the value of the true population mean μ

Finding $+z_{\alpha/2}$

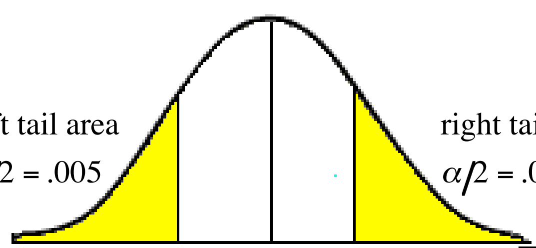
$\alpha = .01$

Confidence Level $= 1 - \alpha$

99% Confidence Level

left tail area

$\alpha/2 = .005$



right tail area

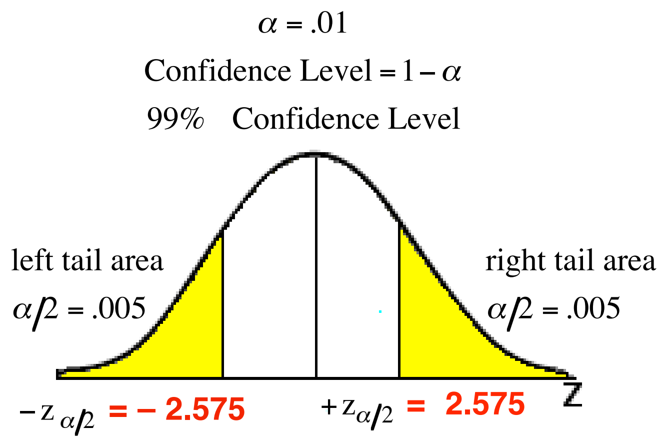
$\alpha/2 = .005$

$-z_{\alpha/2}$ = – 2.575 $+z_{\alpha/2}$ = 2.575 z

Example 3 detailed explanation for finding $+z_{\alpha/2}$

Finding the positive Z score

for an area of $\alpha = .01$ that is equally divided between the left and the right tail $\alpha/2 = .005$ in the left tail and $\alpha/2 = .005$ in the right tail.



Find an area in the **body of the table** that is as close to .005 as possible

The cells at the bottom of the table says for an area of .005 use Z = –2.575

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
Z scores of –3.5 or less use .0001					**AREA**	**Z Score**		**AREA**	**Z Score**	
					0.0500	–1.645		0.0050	**–2.575**	

The positive $+z_{\alpha/2} = |-z_{\alpha/2}|$

The positive $+z_{\alpha/2}$ **value is Z = |– 2.575| = 2.575**

we can verify this by looking for a positive z score with an .9950 area to the left of $+z_{\alpha/2}$

Positive Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
Z scores of 3.5 or more use .9999					**AREA**	**Z Score**		**AREA**	**Z Score**	
					0.9500	1.645		0.9950	**2.575**	

