Chapter 7–1D: Creating a Confidence Interval for the Population Proportion using a Sample Proportion $\hat{p}$

Example 1

Construct a 90 % Confidence Interval for the **Population Proportion** p **for the s**ample data given below. Round to 2 decimal places.

n = 100 , x = 25, Confidence Level = 90 %

Find $\hat{p}$: $\hat{p} = \frac{x}{n} = \frac{25}{100} = .25$. **Find** $\hat{q}$: $\hat{q} = 1 - \hat{p} = 1 - .25 = .75$

Test: $n \bullet \hat{p} = 100 \bullet .25 = 25 \geq 5$ **and** $n \bullet \hat{q} = 100 \bullet .75 = 75 \geq 5$

If we have a 90% Confidence Level then $\alpha = .10$ and $\alpha/2 = .05$

Find $z_{\alpha/2}$ **with a right tail area of** $\alpha/2 =$ **.05** **The positive critical value** $+z_{\alpha/2}$ **is** **1.645**

Find E: $E = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p} \bullet \hat{q}}{n}} = 1.645 \bullet \sqrt{\frac{.25 \bullet .75}{100}} = .07$

Conf. Interval:

$\hat{p} - E < p < \hat{p} + E$

$.25 - .07 < p < .25 + .07$

$.18 < p < .32$

English Statement.

I am 90 % confident that the true Population Proportion p **lies within the interval** $.18 < p < .32$

OR

I am 90 % confident that between 18 % and 32 % of the population has the listed characteristic.

How $z_{\alpha/2}$ was calculated for Example 1

Find the Right Tail Critical Value $z_{\alpha/2}$ with a right tail area of $\alpha/2 =$ 0.05

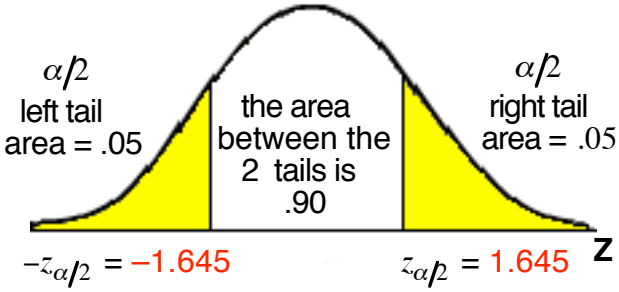
The negative critical value $-z_{\alpha/2}$ is – 1.645

The positive critical value $+z_{\alpha/2}$ is equal to $\left|-z_{\alpha/2}\right|$

The positive critical value $+z_{\alpha/2}$ is 1.645

the total area for both tails is .10

If $\alpha = .10$ then $\alpha/2 = .05$



negative Z critical value for a left tail area of .05

positive Z critical value for a right tail area of .05

Negative Z Scores									
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z									
Z scores	**of –3.5 or less use .0001**				**AREA**	**Z Score**		**AREA**	**Z Score**
					0.0500	–1.645		0.0050	–2.575

Example 2

Construct a Confidence Interval for the sample data and confidence level given below. Round to 2 decimal places.

n = 234 , x = 27, Confidence Level = 95 %

Find $\hat{p}$: $\hat{p} = \frac{x}{n} = \frac{27}{234} = .12$. **Find** $\hat{q}$: $\hat{q} = 1 - \hat{p} = 1 - .12 = .88$

Test: $n \bullet \hat{p} = 234 \bullet .12 = 28.08 \geq 5$ **and** $n \bullet \hat{q} = 234 \bullet .88 = 205.92 \geq 5$

If we have a 95% Confidence Level then $\alpha = .05$ and $\alpha/2 = .025$

Find $z_{\alpha/2}$ **with a right tail area of** $\alpha/2 =$ **.025** **The positive critical value** $+z_{\alpha/2}$ **is 1.96**

Find E: $E = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p} \bullet \hat{q}}{n}} = 1.96 \bullet \sqrt{\frac{.12 \bullet .88}{234}} =$ **.04**

Conf. Interval:

$\hat{p} - E < p < \hat{p} + E$

$.12 - .04 < p < .12 + .04$

$.08 < p < .16$

I am 95 % confident that the true Population Proportion p **lies within the interval** $.08 < p < .16$

English Statement.

I am 95 % confident that between 8 % and 16 % of the population has the listed characteristic.

How $z_{\alpha/2}$ was calculated for Example 2

The total area for both tails is $\alpha = .05$ If $\alpha = .05$ then $\alpha/2 =$ **.025**

Find the Left Tail Critical Value $-z_{\alpha/2}$ with a left tail area of $\alpha/2 =$ .025

The Negative Z Scores Table is used to find the z score with an area of .05 to its left.

the number at the intersection of the –1.9 row and the .06 column is . 025

The tail area to the left of z = – 1.96 is .025

$-z_{\alpha/2} = -1.96$

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233

Find the Right Tail Critical Value $z_{\alpha/2}$ with a right tail area of $\alpha/2 =$ .025

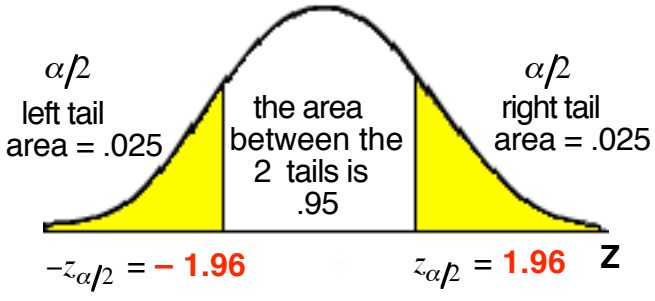
The negative critical value $-z_{\alpha/2}$ is – 1.96

The positive critical value $+z_{\alpha/2}$ is equal to $\left|-z_{\alpha/2}\right|$

The positive critical value $+z_{\alpha/2}$ is 1.96

the total area for both tails is $\alpha = .05$

If $\alpha = .05$ then $\alpha/2 = .025$



$-z_{\alpha/2} = -1.96$ **negative Z critical value for a left tail area of .025**

$z_{\alpha/2} = 1.96$ **positive Z critical value for a right tail area of .025**

Example 3

A researcher at Folsom Lake College samples 120 students at random and finds that 86 of the 120 students sampled own a 3G wireless cell phone. Construct a 92% confidence interval for the true population proportion students at FLC that own a 3G wireless cell phone. Round to 2 decimal places.

n = 120 , x = 86, Confidence Level = 92%

Find $\hat{p}$: $\hat{p} = \frac{x}{n} = \frac{86}{120} = .72$. **Find** $\hat{q}$: $\hat{q} = 1 - \hat{p} = 1 - .72 = .28$

Test: $n \bullet \hat{p} = 120 \bullet .72 = 86.4 \geq 5$ **and** $n \bullet \hat{q} = 120 \bullet .28 = 33.6 \geq 5$

If we have a 92% Confidence Level then $\alpha = .08$ and $\alpha/2 = .04$

Find $z_{\alpha/2}$ **with a right tail area of** $\alpha/2 = .04$ **The positive critical value** $+z_{\alpha/2}$ **is** 1.75

Find E: $E = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p} \bullet \hat{q}}{n}} = 1.75 \bullet \sqrt{\frac{.72 \bullet .28}{120}} = .07$

Conf. Interval:

$\hat{p} - E < p < \hat{p} + E$

$.72 - .07 < p < .72 + .07$

$.65 < p < .79$

I am 92 % confident that the true Population Proportion p **lies within the interval** $.65 < p < .79$

English Statement.

I am 92 % confident that between 65 % and 79 % of the students at FLC own a 3G wireless cell phone.

How $z_{\alpha/2}$ was calculated for Example 3

The total area for both tails is $\alpha = .08$ If $\alpha = .08$ then $\alpha/2 =$ **.04**

Find the Left Tail Critical Value $-z_{\alpha/2}$ with a left tail area of $\alpha/2 =$ .04

The Negative Z Scores Table is used to find the z score with an area of .04 to its left.

Find a number in the body of the table as close to .0400 as possible (in yellow)

The body of the table has a .0401 which is as close to .0400 as possible

the number at the intersection of the –1.7 row and the .05 column is .0401

$$-z_{\alpha/2} = -1.75$$

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367

Find the Right Tail Critical Value $z_{\alpha/2}$ with a right tail area of $\alpha/2 =$ .04

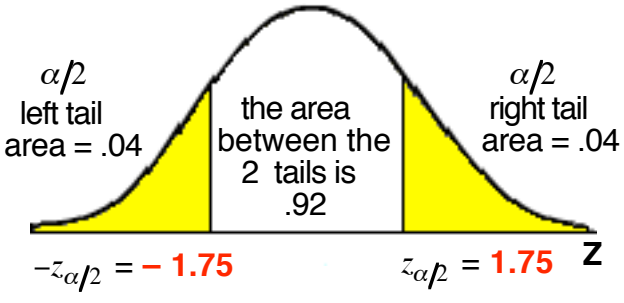
The negative critical value $-z_{\alpha/2}$ is –1.75

The positive critical value $+z_{\alpha/2}$ is equal to $\left|-z_{\alpha/2}\right|$

The positive critical value $+z_{\alpha/2}$ is 1.75

the total area for both tails is $\alpha = .08$

If $\alpha = .08$ then $\alpha/2 = .04$



negative Z critical value for a left tail area of .04

positive Z critical value for a right tail area of .04

Example 4

The Folsom Auto Mall conducted a survey regarding Folsom residents plans to buy a new car in the next 12 months. The research found that of the 289 residents surveyed 21 planned on buying a new car in the next 12 months. Construct a 99 % confidence interval for the true population proportion of Folsom residents who plan to buy a new car in the next 12 months.

n = 289 , x = 21, Confidence Level = 99%

Find $\hat{p}$: $\hat{p} = \frac{x}{n} = \frac{21}{289} = .07$ **Find** $\hat{q}$: $\hat{q} = 1 - \hat{p} = 1 - .07 = .93$

Test: $n \bullet \hat{p} = 289 \bullet .07 = 20.23 \geq 5$ **and** $n \bullet \hat{q} = 289 \bullet .93 = 268.77 \geq 5$

If we have a 99 % Confidence Level then $\alpha = .01$ and $\alpha/2 = .005$

Find $z_{\alpha/2}$ **with a right tail area of** $\alpha/2 =$ **.005** **The positive critical value** $+z_{\alpha/2}$ **is 2.575**

Find E: $E = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p} \bullet \hat{q}}{n}} = 2.575 \bullet \sqrt{\frac{.07 \bullet .93}{289}} =$ **.04**

Conf. Interval:

$\hat{p} - E < p < \hat{p} + E$

$.07 - .04 < p < .07 + .04$

$.03 < p < .11$

I am 99 % confident that the true Population Proportion p **lies within the interval** $.03 < p < .11$

English Statement.

I am 99 % confident that between 3 % and 11 % of Folsom residents plan to buy a new car in the next 12 months.

How $z_{\alpha/2}$ was calculated for Example 4

The total area for both tails is $\alpha = .01$ If $\alpha = .01$ then $\alpha/2 =$ **.005**

Find the Left Tail Critical Value $-z_{\alpha/2}$ with a left tail area of $\alpha/2 =$ .005

The Negative Z Scores Table is used to find the z score with an area of .005 to its left.

The cells at the bottom of the z table say to use Z = – 2.575 for a left tail area of .005

$-z_{\alpha/2}$ = – 2.575

Negative Z Scores										
Standard Normal (Z) Distribution: Cumulative Area to the LEFT of Z										
Z scores	**of –3.5 or less use .0001**				**AREA**	**Z Score**		**AREA**	**Z Score**	
					0.0500	–1.645		0.0050	–2.575	

Find the Right Tail Critical Value $z_{\alpha/2}$ with a right tail area of $\alpha/2 =$ .005

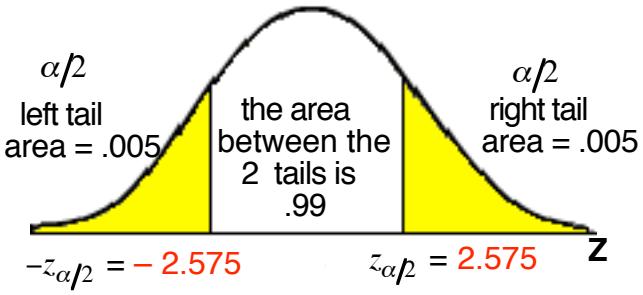
The negative critical value $-z_{\alpha/2}$ is – 2.575

The positive critical value $+z_{\alpha/2}$ is equal to $\left|-z_{\alpha/2}\right|$

The positive critical value $+z_{\alpha/2}$ is 2.575

the total area for both tails is $\alpha = .01$

If $\alpha = .01$ then $\alpha/2 = .005$



negative Z critical value for a left tail area of .005

positive Z critical value for a right tail area of .005

