

Section 5 – 3 The Mean and Standard Deviation of a Binomial Distribution

Previous sections required that you to find the Mean and Standard Deviation of a Binomial Distribution by using the values from a table. This section will use a formula to find the Mean and Standard Deviation of a Binomial Distribution **without creating the table.**

The Population Mean and Population Standard Deviation Formulas for a Binomial Distribution if the sample size n is given and the probability of success p is known

The Population Mean

$$\mu_x = n \cdot p$$

The Population Standard Deviation

$$\sigma_x = \sqrt{n \cdot p \cdot q}$$

Example 1

Assume each of the following problems yield a **binomial distribution** with n trials and the probability of success for one trial is p. Find the Mean and the Standard Deviation. Round to 2 decimal places.

$$n = 40 \quad p = .30$$

$$\text{Find } q: q = 1 - .30 = .70$$

$$n = 40 \quad p = .30 \quad q = .70$$

$$\mu_x = n \cdot p = 40 \cdot .30 = 12$$

$$\mu_x = \sqrt{n \cdot p \cdot q} = \sqrt{40 \cdot .3 \cdot .7} \approx 2.90$$

The formula for the mean makes sense. $P(x) = .30$ says that you have a 30% chance of picking something with 1 pick. If you make 40 picks you would expect to get 30% of 40 or 12 as an average outcome. The formula for the Standard Deviation is less straight forward and the algebraic proof will be omitted.

Example 2:

Ten percent of all students are left handed. Find the mean and standard deviation for the number of left handed students in a class of 30 students. Round to 2 decimal places.

$$n = 30 \quad p = .10$$

$$\text{Find } q: q = 1 - .10 = .90$$

$$n = 30 \quad p = .10 \quad q = .90$$

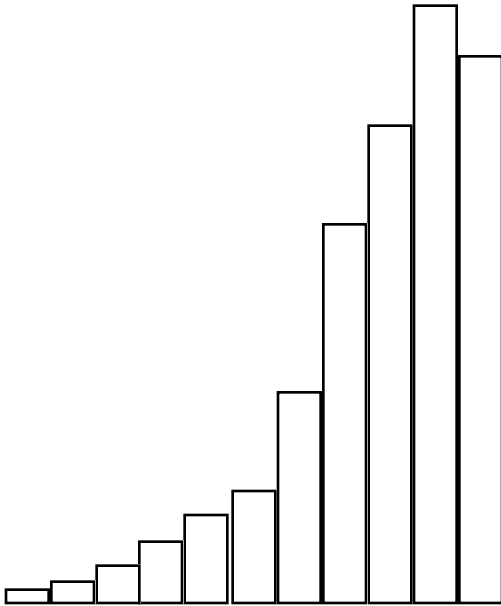
$$\mu_x = n \cdot p = 30 \cdot .10 = 3.00$$

$$\mu_x = \sqrt{n \cdot p \cdot q} = \sqrt{30 \cdot .1 \cdot .9} \approx 1.64$$

Is the Binomial Probability Distribution a Bell Shaped Distribution?

In general, a Binomial Probability Distribution **IS NOT** a Bell Shaped Distribution. As the number of trials (n) becomes larger or if the value of p becomes closer to .5 then the distribution becomes more bell shaped

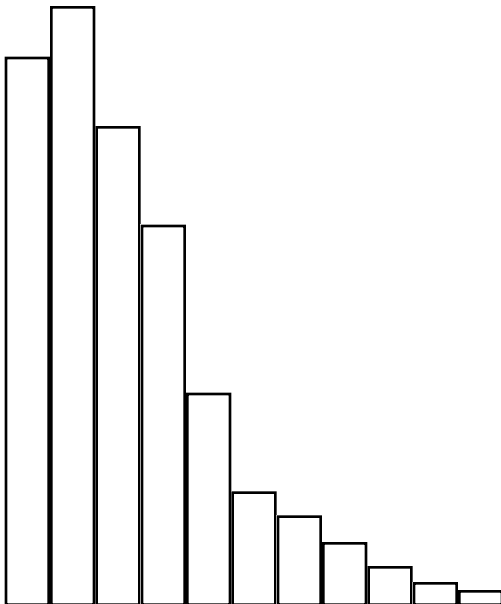
If $p > .5$ then the distribution is Skewed Left (Long Tail to the Left)



x	P(x)
1	.01
2	.02
3	.04
4	.08
5	.10
6	.12
7	.18
8	.25
9	.20

If the probability of a success is more than .5 ($p > .5$) the distribution is **skewed left**. A distribution that is skewed has the smallest probabilities and lowest rectangles on the left side of a probability histogram. It has the smaller values for $P(X)$ at the top of the probability distribution table.

If $p < .5$ then the distribution is Skewed Right (Long Tail to the Right)

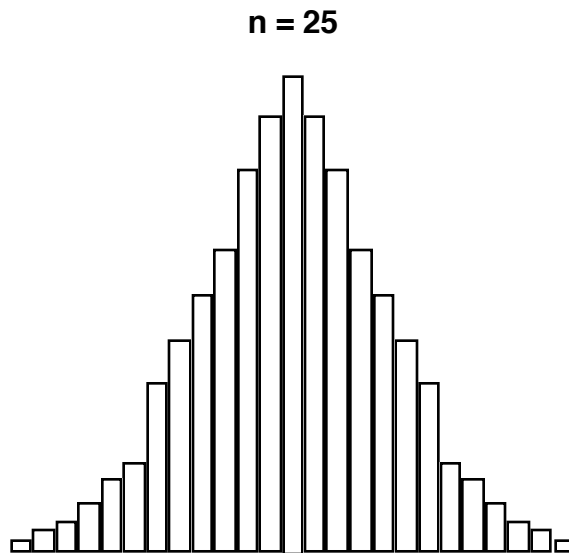
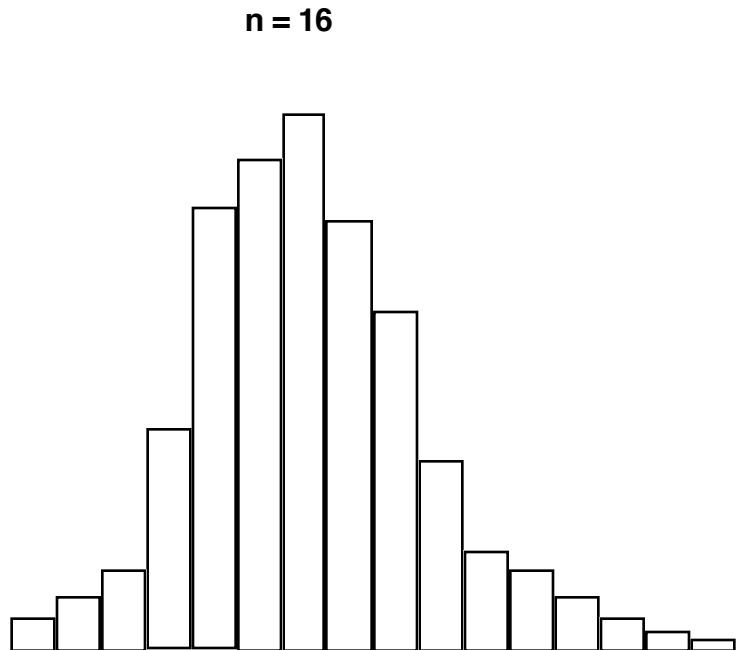
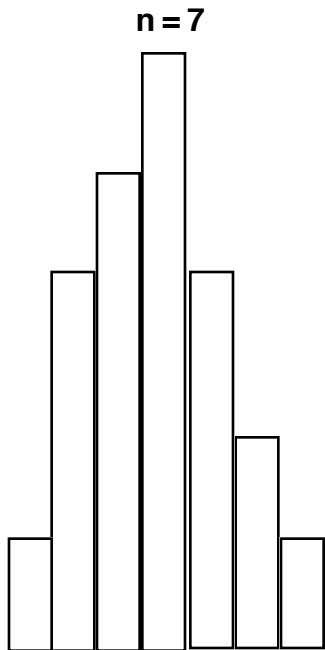


x	P(x)
1	.20
2	.25
3	.18
4	.12
5	.10
6	.08
7	.04
8	.02
9	.01

If the probability of a success is less than .5 ($p < .5$) the distribution is **skewed right**. A distribution that is skewed has the smallest probabilities and lowest rectangles on the right side of a probability histogram. It has the smaller values for $P(X)$ at the bottom of the probability distribution table.

Binomial Distributions with a fixed value for p

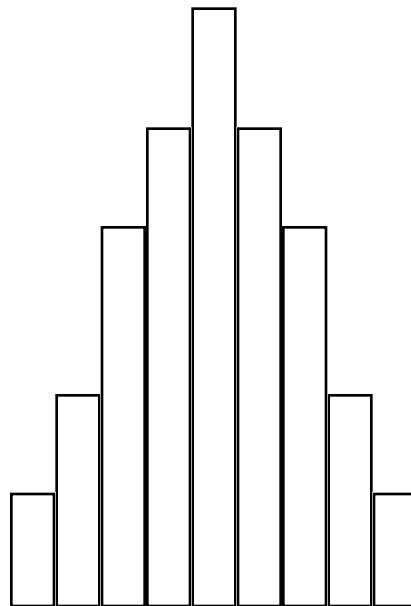
For any given p
as n gets larger the Binomial Distribution gets **more bell shaped**



When Is the Binomial Probability Distribution a Bell Shaped Distribution?

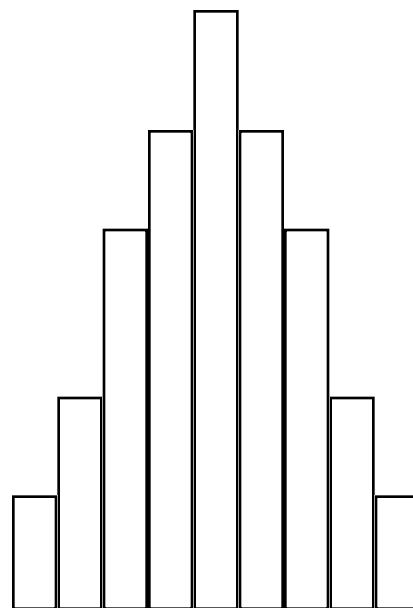
If the probability of a success is $.5$ ($p = .5$)
then the distribution is exactly bell shaped or normal.

x	P(x)
1	.02
2	.08
3	.13
4	.17
5	.20
6	.17
7	.13
8	.08
9	.02



If **both** $n \cdot p \geq 5$ **AND** $n \cdot q \geq 5$ **are true**
then the Binomial Distribution is not bell shaped
but the distribution can be considered **close enough**
to bell shaped to be considered bell shaped.

x	P(x)
1	.02
2	.08
3	.13
4	.16
5	.21
6	.18
7	.11
8	.07
9	.04



If **both** $n \cdot p \geq 5$ **AND** $n \cdot q \geq 5$ **are true**
than the distribution can be considered
close enough to bell shaped to be
considered bell shaped.

When Is the Binomial Probability Distribution a Bell Shaped Distribution?

The Binomial Probability Distribution can be considered **approximately** bell shaped if

$$p = .5$$

or if

$$n \cdot p \geq 5 \text{ AND } n \cdot q \geq 5 \text{ are both true}$$

The requirement that $n \cdot p \geq 5$ **AND** $n \cdot q \geq 5$ for the distribution to be bell shaped (normal) while very common is not standard. Some textbooks require that $np \geq 10$ and $nq \geq 10$. Others chose different tests. The Michael Sullivan Statistics book uses the test $n \cdot p \cdot q \geq 10$ for the Binomial Probability Distribution to be considered approximately bell shaped and quotes "P. H. Ramsey: *Journal Of Educational Statistics* 1998 V-13"

Since the Binomial Probability Distribution is not Bell Shaped unless $p = .5$ any test is based on how close to Bell Shaped you are satisfied with.

Example 1

$$n = 60 \quad p = .60 \quad q = .40$$

$$n \cdot p = 36 \quad n \cdot q = 24$$

Is the Binomial Distribution Bell Shaped?

Yes $n \cdot p \geq 5$ **AND** $n \cdot q \geq 5$ **are both true**

Example 2

$$n = 4 \quad p = .5 \quad q = .5$$

Is the Binomial Distribution Bell Shaped?

Yes ($p = .5$)

Example 3

$$n = 24 \quad p = .8 \quad q = .2$$

$$n \cdot p = 19.2 \quad n \cdot q = 4.8$$

Is the Binomial Distribution Bell Shaped?

No $n \cdot p \geq 5$ **AND** $n \cdot q \geq 5$ **are not both true**

Unusual values for a Binomial Probability Distribution

A Binomial Probability Distribution is Bell Shaped (Normal) if

$$p = .5$$

OR

if $n \cdot p \geq 5$ and $n \cdot q \geq 5$ are both true

For Binomial Probability Distributions that are Bell Shaped

The range of usual values of x are the x values

within 2 standard deviations on either side of the mean

$$\mathbf{x \text{ is usual if } \mu_x - 2\sigma_x \leq x \leq \mu_x + 2\sigma_x}$$

Unusual values are more than 2 standard deviations to either side of the mean

$$\mathbf{x \text{ is unusual if } x < \mu_x - 2\sigma_x \text{ or } x > \mu_x + 2\sigma_x}$$

Note: We **do not** compute Unusual or Usual Values for distributions that are not bell shaped.

Example 1

30 percent of all students at FLC live north of the river. Find the mean and standard deviation for the number of students who live north of the river in a class of 60 students. Round to 2 decimal places.

$n = 60$ $p = .30$ Find $\mu_x =$ _____ Find $\sigma_x =$ _____

$n \cdot p =$ _____ $n \cdot q =$ _____ Is the Binomial Distribution Bell Shaped? _____

If the Binomial Distribution is Bell Shaped what is the range of usual values?

Would it be unusual to have a class of 60 students have 17 students that live north of the river?

Would it be unusual to have a class of 60 students have 2 students that live north of the river?

Solution:

Find q: $q = 1 - .30 = .70$

$n = 60$ $p = .30$ $q = .70$

Mean: $\mu_x = n \cdot p = 60 \cdot .30 = 18$

SD: $\mu_x = \sqrt{n \cdot p \cdot q} = \sqrt{60 \cdot .3 \cdot .7} \approx 3.55$

$n \cdot p = 18$ $n \cdot q = 42$ Is the Binomial Distribution Bell Shaped? **Yes** ($np \geq 5$ **AND** $nq \geq 5$)

x is usual if $\mu_x - 2\sigma_x \leq x \leq \mu_x + 2\sigma_x$

$10 - 2(3.55) \leq x \leq 10 + 2(3.55)$

$2.9 \leq x \leq 17.1$

Would it be unusual in a class of 60 students to have 17 students that live north of the river? **NO**

Would it be unusual in a class of 60 students to have 2 students that live north of the river? **Yes**

Example 2

A basketball player makes 60% of her free throws. Find the mean and standard deviation for the number of free throws made if she shoots 150 free throws. Round to 2 decimal places.

$n = 150$ $p = .60$ Find $\mu_x =$ _____ Find $\sigma_x =$ _____

$n \cdot p =$ _____ $n \cdot q =$ _____ Is the Binomial Distribution Bell Shaped? _____

If the Binomial Distribution is Bell Shaped what is the ranges of usual values?

Would it be unusual for the player to make 115 out of 150 free throws?

Would it be unusual for the player to make 80 out of 150 free throws?

Solution:

Find q: $q = 1 - .60 = .40$

$n = 150$ $p = .60$ $q = .40$

Mean: $\mu_x = n \cdot p = 150 \cdot .60 = 90$

SD: $\mu_x = \sqrt{n \cdot p \cdot q} = \sqrt{150 \cdot .6 \cdot .4} = 6$

$n \cdot p = 90$ $n \cdot q = 60$ Is the Binomial Distribution Bell Shaped? **Yes** ($np \geq 5$ **AND** $nq \geq 5$)

x is usual if $\mu_x - 2\sigma_x \leq x \leq \mu_x + 2\sigma_x$

$$90 - 2(6) \leq x \leq 90 + 2(6)$$

$$78 \leq x \leq 102$$

Would it be unusual for the player to make 115 out of 150 free throws? Yes

Would it be unusual for the player to make 80 out of 150 free throws? No

Example 3

Mar's Company states that 14% of its M&M candies are colored yellow. Find the mean and standard deviation for the number of yellow M&M's in a bag with 30 M&M's. Round to 2 decimal places.

$n = 30$ $p = .14$ Find $\mu_x =$ _____ Find $\sigma_x =$ _____

$n \cdot p =$ _____ $n \cdot q =$ _____ Is the Binomial Distribution Bell Shaped? _____

If the Binomial Distribution is Bell Shaped what is the ranges of usual values?

Would it be unusual for a bag of 30 M&M's to have 24 yellow M&M's?

Would it be unusual for a bag of 30 M&M's to have 6 yellow M&M's?

Solution:

Find q: $q = 1 - .14 = .86$

$n = 30$ $p = .14$ $q = .86$

Mean: $\mu_x = n \cdot p = 30 \cdot .14 = 4.2$

SD: $\mu_x = \sqrt{n \cdot p \cdot q} = \sqrt{30 \cdot .14 \cdot .86} \approx 1.90$

$n \cdot p = 4.2$ $n \cdot q = 25.8$ Is the Binomial Distribution Bell Shaped? **NO**

neither $p = 5$ OR ($np \geq 5$ AND $nq \geq 5$) are TRUE

We cannot find usual or unusual values in a distribution that is not normal (or approximately normal)