Section 5 – 1D: **Probability Distribution Tables**

Expected Value

The Expected Value of a Probability Distribution is denoted by E.
It represents the average value of all the outcomes.

A person at a horse race bets 5 dollars on each race. If he picks the correct horse to win the track keeps the 5 dollar bet and pays out 12 dollars for a **7 dollar gain** (Win +7). If he picks the wrong horse to win the track keeps the 5 dollars and the bettor has a **5 dollar loss** (Lose –5).

The list below shows the outcome for 20 races.

Lose –5, Lose –5, Lose –5, **Win + 7**, Lose –5, Lose –5, Lose –5, Lose –5, Lose –5

Lose –5, Lose –5, Lose –5, **Win + 7,** Lose –5, Lose –5, Lose –5, Lose –5, Lose –5,

Lose –5, **Win + 7,**

Compute the money won.

The bettor won 7 dollars 3 times in 20 bets. He won **21 dollars** on the 20 bets. $3 \bullet +7 = 21$

Compute the money lost

The better lost 5 dollars 17 times in 20 bets. He lost **85 dollars** on 20 bets. $-5 \bullet +17 = -85$

Expected Value

Compute the Average Outcome Per one bet.

The bettor won **21 dollars** and lost **85 dollars** on the 20 bets. This is a net loss of 64 dollars (–64) on the 20 bets. If the **loss of –64 is divided by the total number of bets** the results will be the **average outcome per bet**. –64/ 20 = – 3.20 This is a **loss of $ 3.20 per bet.**

This average outcome is called the **Expected Value (E) of each 5 dollar bet.** The Expected Value of –3.20 means that for every 5 dollar bet the bettor can expect to lose $ 3.20. No single bet has an outcome of –3.20 but **the average of all the 7 dollar wins and 5 dollar loses is a loss of 3.20 (–3.20) for every 5 dollar bet if a large numbers of bets are made.**

If a bettor places 10 bets then they can expect to lose 32 dollars. This will not happen in every 10 bets but as the number of bets increases we would would expect the 3.20 loss to be the average outcome of each 5 dollar bet.

Expected Value

The Expected Value of a Probability Distribution is denoted by E.
E represents the average value of all the outcomes.
E is the Population Mean for a Probability Distribution

The Mean of a Discrete Probability Distribution was defined to be

$$\mu_x = \sum[x \bullet P(x)]$$

so we can state that the **Expected Value E** of the Discrete Probability Distribution is

$$\mathrm{E} = \mu_x = \sum[x \bullet P(x)]$$

A Probability Distribution Table for finding the Expected Value E based on Probabilities

The random variable x stands for how much is won or lost on **a single outcome**.

If the outcome is a win than **the value of x is how much you GAIN from a win on one outcome.** This will be a positive number. Be sure to **deduct the cost of playing** one time from the winning amount to get the **Net Gain.**

If the outcome is a loss than **the value of x is how much you lose on one outcome.** This will be a negative number. This is normally the cost of playing one time.

	Outcome	**x**	**P(X)**	$x \bullet \mathrm{P}(x)$
Win	written description of **what a win is**	**Net amount gained on one outcome win – cost (positive)**	P(win) will be given	calculate $x \bullet \mathrm{P(win)}$ for this row
Lose	written description of **what a loss is**	**Amount lost on one outcome (negative)**	P(loss) = 1– P(win)	calculate $x \bullet \mathrm{P(lose)}$ for this row
			E =	the total of the 2 numbers above

Finding E with the use of a Probability Distribution Table

Example 1:

A island cruse costs $ 500 which you must pay in advance. You may also buy a cruse trip cancelation policy that costs $ 30. If the you cannot take the cruse due to your illness or death you receive a check back for the $ 500 cost of the trip. The probability of a trip being canceled is .05 P(you cancel) = .05 What is the expected value **for the traveler** who buys trip Insurance?

Step 1. Describe the winning event (you cancel trip) and the losing event (you do not cancel trip)

Step 2. List the outcome if the trip is canceled: win x = **500 – 30 cost of insurance** = **+ 470**
List the outcome if the trip is not canceled: loss x = **– 30** (lost 30 dollars)

Step 3: List the probability for a win P(you have to cancel) = .05 and loss P(not canceled) = .95

Step 4. Write the product $x \bullet P(\text{you cancel})$ for the win row + 470 • .05 = **23.50**
Write the product $x \bullet P(\text{you do not cancel})$ for the the lose row – 30 • .95 = **– 28.50**

Step 5. The Expected Value E is found by adding the two values in the $x \bullet P(x)$ column. Be sure to include a sign with the expected value (23.50 – 28.50 = **–5.00**)

	Event	outcome x	P(x)	$x \bullet \text{P}(x)$
Win	You have to cancel the trip	**+ 470**	.05	**23.50**
Lose	You do not have to cancel the trip	**– 30**	.95	**– 28.50**
Expected Value = **E** = $\sum[x \bullet P(x)] =$				**– 5.00**

The Expected Value for each $ 30 policy that a **traveler buys** is E = **–5.00**

The Expected Value for each $ 30 policy the **Insurance company** sells is +5.00

The average **passenger who buys a policy will lose $ 5** and the **Insurance company will make $ 5** for each policy that it sells.

Example 2:

A roulette has 18 green trays, 18 red trays , and 1 white tray for the ball to land in. The casino takes your bet of **$ 5.00** that the ball will land in a green tray. The casino will pay you $ 10.00 if the ball lands on the color green. The probability of winning by betting on green is $\frac{18}{37}$. What is the expected value **for the bettor**?

Step 1. Describe the winning event (lands on green) and the losing event (does **not** land on green)

Step 2. List the outcome for a win: x = 10 dollar win – 5 dollar cost of the bet = **+ 5**
List the outcome for a loss x = **– 5** (lost 5 dollars)

Step 3: List the probability for a win P(lands on green) = $\frac{18}{37}$

and loss P(does not land on green) = $\frac{19}{37}$

Step 4. Write the product $x \bullet P(x)$ for the win row $5 \bullet \frac{18}{37}$ = **2.43**

and the loss row $-5 \bullet \frac{19}{37}$ = **– 2.57**

Note: Do not change 18/ 37 into a decimal. The will create a round off error.

Step 5. The Expected Value E is found by adding the two values in the $x \bullet P(x)$ column. Be sure to include a sign with the expected value (**2.43** **– 2.57** = **–.14**)

	Event	outcome x	P(x)	$x \bullet \mathrm{P}(x)$
Win	Lands on Green	**+ 5**	18/ 37	**2.43**
Lose	Does not land on Green	**– 5**	19/ 37	**– 2.57**
Expected Value = **E** = $\sum[x \bullet P(x)] =$				**– 0.14**

The Expected Value f**or the bettor** for each $ 5 bet on Green is E = – 0.14

The Expected Value **for the casino** for each $ 5 bet on Green is E = + 0.14

The average player who bets $ 5 on Green will lose 14 cents and the casino will make 14 cents.

Note: DO NOT round off $\frac{18}{37}$ to a decimal (.49) and then use that number to find the value of $5 \bullet \frac{18}{37}$
Using that rounded off number for P(x) to find $x \bullet \mathrm{P}(x)$ that you then round off increases the error.