Section 5 – 1A : Discrete Probability Distributions Introduction

Discrete Probability Distribution Tables

A probability distribution table is **like the relative frequency tables** that we constructed in chapter 2. Those relative frequency tables had all the possible values of x in the left column and the right column contained the percent (as a decimal) that each outcome occurred out of the total number of outcomes.

The probability distribution table has the same format.
The left column will contain a list of all the the possible values of the random variable x.
The right column will contain the probability that each of the possible x values occurs.

A Relative Frequency Table

x: Number Of Girls in 3 births	Relative Frequency(x)
0	.13
1	.38
2	.38
3	.13

The **left column** will contain a list of **all the possible outcomes of the variable x.**
The **right column** will contain the **proportion of the population that each of the possible outcomes occurs.**

A Probability Distribution Table

x: Number Of Girls in 3 births	P(x)
0	.13
1	.38
2	.38
3	.13

The **left column** will contain a list of **all the possible outcomes of the variable x.**
The **right column** will contain the **probability that a given value of x will occur.**

Example 4A

.13 (13%) of the women who have 3 children have 0 girls

.38 (38%) of the women who have 3 children have 2 girls

Example 4 B

The probability that if I select 1 women from the population of women who have 3 children that they will have 0 girls is .13 (13%)

The probability that if I select 1 women from the population of women who have 3 children that they will have 2 girls is .38 (38%)

It is easy to see that if 13% of a population has a given attribute, then there is a 13% chance that a person selected from that population will have that attribute.

The Random Variable x:

A **Random Variable** is a variable (typically represented by x) whose **different numerical values** describe **all the possible outcomes** from a procedure or experiment.

A Discrete Random Variable: A random variable is discrete if the possible values for the variable are **countable**. For the purposes of this chapter we will use whole numbers for the possible values of the Discrete Random Variable. That is, the possible values for x are 0, 1, 2, 3, 4, ...

Continuous Random Variable: A random variable is continuous if the values are associated with **measurements on a continuous scale.** Measurements of time, volume, weight , temperature and distance are examples of of such measurements.

This unit will only deal with Discrete Random Variables. We will work with continuous random variables in later chapters.

Examples of a Discrete Random Variable x:

Example 1

x is the number of heads that occurred when a coin was flipped **4 times in a row.**

x is a **discrete** variable because the **possible values for x** are x = 0 , 1, 2 , 3 or 4.

x is **random** variable because the value of x will vary with each new set of 4 flips

Example 2

x is how many of the **6 patients** that used a new drug showed an improvement.

x is a **discrete** variable because **the possible values for x** are x = 0 ,1, 2 , 3 , 4 , 5 or 6

x is a **random** variable because the value of x will vary with each new set of 6 patients tested.

Example 3

5 students who attend FLC are asked how many days a week they go to school.

x is a **discrete** variable because **the possible values for x are** x = 0 , 1, 2 , 3 , 4 , 5

x is **random** variable because the value of x will vary with each new set of 5 students.

There are two types of Discrete Probability Distributions Tables

One type is based on a Sample and
the other is based on a Theoretical Model

The outcome from flipping a coin 25 times

H H H H H H H H H H H = **11 H**

T T T T T T T T T T T T T T = **14 T**

A Probability Distribution Table
based on a Sample

The outcomes from
a sample of 25 coin flips

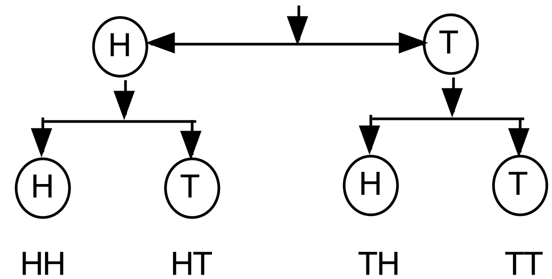
Heads = 11 Tails = 14

x: **Number Of Heads** in 1 flip of a coin	Freq (x)	P (x)
0	11	$P(0\,H) = \frac{11}{25} = .44$
1	14	$P(1\,H) = \frac{14}{25} = .56$

The values of P(x) will vary based on the sample

The probabilities for each x value in the Probability Table **will vary depending on the sample.** This table **does not reflect the entire population**.

As the **sample size gets larger** the values for P(x) based on the sample will **get closer** to the exact population values of P(x)



A Probability Distribution Table
based on a Theoretical Model

The probability if getting a head on one flip of the coin is 1/ 2

P(H) = 1/ 2 and P(T) = 1/ 2

x: **Number Of Heads** in 1 flip of a coin	P(x)
0	.50
1	.50

The values of P(x) are fixed and reflect the Theoretical population

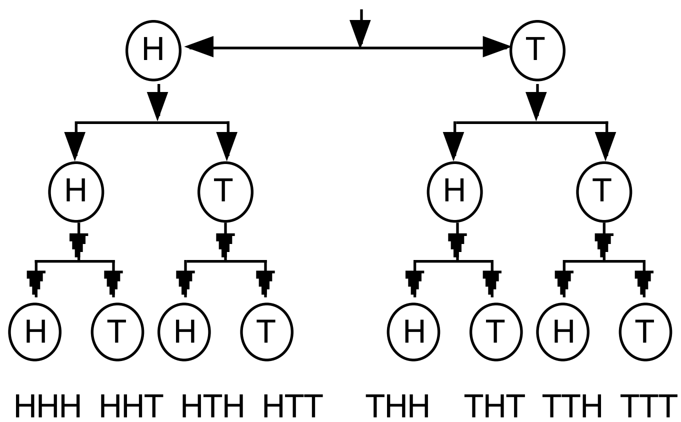
The probabilities for each x value in the Probability Table of the theoretical model **will not vary**. They are **not based on a sample.** These probabilities represent the entire population. We do not need to list the entire population to find each P(x). We use formulas to develop the P(x) values in the table. We can then use this type of table to answer questions about **what probabilities for the variable x can be expected for the entire population.**

Example

Creating a Probability Distribution Table
based on a theoretical model if the probability of each outcome in the same

One way to create a theoretical model of a Probability Distribution Table would be to create the entire population sample space based on the theoretical outcomes and then calculate the value of P(x) for each x as a proportion of the total sample space.   P(H) = 1/ 2 and P(T) = .1/ 2

Procedure: Flip a coin 3 times and record the number of heads.
The model below contains the **population of all 8 equally likely outcomes.**



The random variable x (the number of Heads) can have a value of 0, 1, 2 or 3.
The probability of each of the possible values for x is computed as shown below.

x: Number of Heads in 3 flips	P(x)		How each P(x) was calculated for each x
0	.125		$P(0 \text{ heads}) = \dfrac{\text{number of outcomes with 0 head}}{\text{total number of outcomes}} = \dfrac{1}{8} = .125$
1	.375		$P(1 \text{ heads}) = \dfrac{\text{number of outcomes with 1 head}}{\text{total number of outcomes}} = \dfrac{3}{8} = .375$
2	.375		$P(2 \text{ heads}) = \dfrac{\text{number of outcomes with 2 head}}{\text{total number of outcomes}} = \dfrac{3}{8} = .375$
3	.125		$P(3 \text{ heads}) = \dfrac{\text{number of outcomes with 3 head}}{\text{total number of outcomes}} = \dfrac{1}{8} = .125$

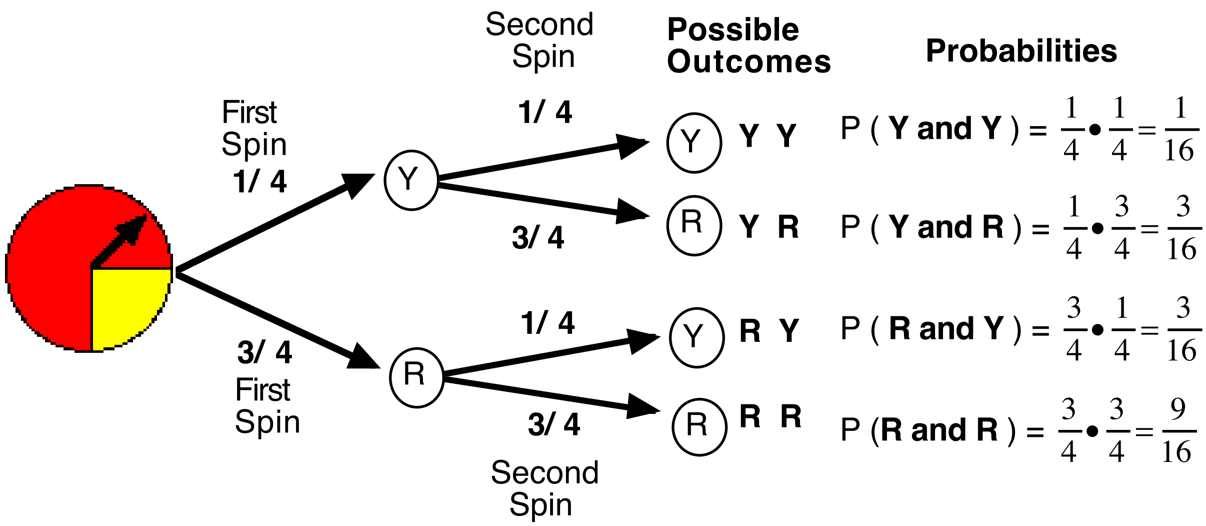
The third column on the right is not normally shown as part of a probability distribution table. It is included here to help understand how each P(x) was calculated. It is also common to state P(x) be a decimal number rounded off to a given number of decimal places. For this chapter we will list all P(X) as a decimal number **rounded off to 2 decimal places** unless stated otherwise.

Example

Creating a Probability Distribution Table
based on a theoretical model if the probability of each outcome is NOT the same

One way to create a theoretical model of a Probability Distribution Table would be to create the entire branching diagram and determine the probability for each outcome $P(A \text{ and } B) = P(A) \bullet P(B)$

Procedure: Spin Spinner 1 and then spin Spinner 1 again.



x: Number of times a red is spun in 2 spins	$P(x)$	$P(x)$
0	$\frac{1}{16}$	$P(0 \text{ Reds}) = P(Y \text{ and } Y) = \frac{1}{16}$
1	$\frac{6}{16}$	$P(1 \text{ Red}) = P(R \text{ and } Y) + P(Y \text{ and } R) = \frac{6}{16}$
2	$\frac{9}{16}$	$P(2 \text{ Reds}) = P(R \text{ and } R) = \frac{9}{16}$

$$P(0 \text{ Red}) + P(1 \text{ Red}) + P(2 \text{ Red}) = \frac{1}{16} + \frac{6}{16} + \frac{9}{16} = \frac{16}{16} = 1$$

The table above represents all the possible outcomes
Since $\sum P(x) = 1$ the table
represents the entire population of probabilities for the discrete variable x.

Creating a Probability Distribution Table based on a theoretical model produces a table that represents the entire population of probabilities for the discrete variable x.

Discrete Probability Distribution Tables
for 2 different samples will not be the same.

Procedure: Flip a coin 3 times and record the number of heads.

The outcome from **a sample of 48 flips**

x: Number 0f heads in 3 flips	Freq. (x) n = 48	$P(x) = \frac{\text{frequency of x}}{\text{total sample size n}}$
0	5	$P(0\,H) = \frac{6}{48} = .104$
1	17	$P(1H) = \frac{17}{48} = .354$
2	19	$P(2\,H) = \frac{19}{48} = .396$
3	7	$P(3H) = \frac{7}{48} = .184$

The outcome from a **sample of 1200 flips**

x: Number 0f heads in 3 flips	Freq. (x) n = 24	$P(x) = \frac{\text{frequency of x}}{\text{total sample size n}}$
0	147	$P(0\,H) = \frac{147}{1200} = .123$
1	452	$P(1H)) = \frac{452}{1200} = .377$
2	446	$P(2\,H) = \frac{446}{1200} = .372$
3	155	$P(3H) = \frac{155}{1200} = .129$

The sample of 48 flips **did not have P(x) values equal to the theoretical model** we developed using the population sample space in Example 6. The values of P(x) for the table based on 1200 samples **was closer to the P(x) values based on the theoretical model** but they were sill not exactly equal to theoretical model. If we conducted this procedure 30,000 times we would expect the values of each P(x) to get even closer to the theoretical model.

The value for P(0) for large samples would be about .125
The value for P(1) for large samples would be about .375
The value for P(2) for large samples would be about .375
The value for P(3) for large samples would be about .125

You may wonder if there is a way to create a theoretical table that world describe the **expected outcomes** based on flipping the 3 coins a very large number of times ($n \to \infty$) without creating the entire population sample space or conducting a large sample.

Yes there is. Under **some conditions** we can construct a **Probability Distribution Table** using probability formulas to find the **expected outcomes.** The values in a table of this type **are considered to be the exact theoretical values** for the population probabilities of the variable x.