Section 4 – 6 C: **Combinations**

(Order DOES NOT Matter)

A **combination** is an **arrangement** **without regard to order** **of n DIFFERENT ITEMS** with **none of the items being REPEATED.**

1. You have n items to chose from. Each of **the n items are different.**
2. You select r items. All **the different orderings of the same items** are counted as the **same combination**. If we change the order of 2 items they are still 1 combination.

The number of **combinations** of **n different items chosen r at a time is**

$$_{n}C_{r}$$

Example

Joe, Bob and Sue are selected to work in a 2 member study group. No matter who is selected first or second they are still the same study group. How many different study groups can there be?

Consider all the possible arraignments of **three people** named Joe , Bob and Sue taken **2 at a time**

there are **6 different Arraignments** of the three people **Bob, Sam and Joe taken two names at a time**

Joe , Bob	**Sue , Joe**	**Bob , Joe**
Joe , Sue	**Sue , Bob**	**Bob , Sue**

Looking at the 6 **Arraignments** above we can see that some of them contai**n the same names in a different order.** All **the different orderings of the same item**s are counted as the **same combination**.

Joe , Bob and **Bob , Joe** both contain Joe and Bob **but in a different order so they are counted as 1 combination**

Joe , Sue and **Sue , Joe** both contain Sue and Joe **but in a different order so they are counted as 1 combination**

Sue , Bob and **Bob , Sue** both contain Sue and Bob **but in a different order so they are counted as 1 combination**

Joe Bob = Bob Joe **Joe Sue = Sue Joe** **Sue Bob = Bob Sue**

There are **3 different combinations** of the three people **Bob, Sam and Joe taken two names at a time**

The number of **combinations** of **3 items chosen 2 at a time is**

$_3C_2 = 3$

Using the Calculator to find the value of $_nC_r$

1. Clear the screen with the **CLEAR** key
2. Press the number keys to display the value of **n**
3. Push the PRB key to get the 3 probability selections **$_nP_r$ $_nC_r$!**
4. Press the Right Arrow key ▶ to get the underline under the $_nC_r$ symbol.
5. Press the ENTER = key to select the $_nC_r$ symbol.
6. Press the number keys to input the value of **r**
7. A display like $3 _nC_r 2$ will be shown.
8. To find the value of $3 _nC_r 2$ press the ENTER = key.

Example 1

$_{14}C_5 = 2,0020$

Example 2

$_{12}C_4 = 495$

Example 3

$_9C_5 = 126$

Example 4

In how many different ways can **10 golfers** be selected to play on a **4 member team** in a round of golf?

Solution: We have 10 **distinct people** and we are **selecting 4 of them**. It does not matter what order the players are selected in. They all are just team members on the same team.

$n = 10$ and $r = 4$

$$_{10}C_4 = 210$$

Example 5

In how many different ways can **9 people** be selected to serve on a 3 member clean up committee for a local charity dinner?

Solution: We have 9 distinct people and we are selecting 3 of them to serve on a clean up committee. It does not matter what order the people are selected in. They all are just members on the same committee.

n = 9 and r = 3

$_9C_3 = 84$

Example 6

A state lottery has 49 balls in a container numbered from 1 to 49. 6 of the balls are chosen at random and the 6 numbers are listed. The winner must have all 6 of the numbers. How many different arrangements of 6 winning numbers be made?

Solution: We have 49 distinct numbers and we are selecting 6 of them. It does not matter what order the numbers are selected in. They all are just part of the 6 winning numbers.

The number of combinations of **49 distinct items chosen 6 at a time without regard to order is**

$$_{49}C_{6} = 13{,}983{,}816$$

There are 13,983,816 possible winning numbers for this lottery. The probability of selecting a winning number is about 1 in 14 million.

Example 7

A doctor wants to test a new drug on 15 patients. 20 patients volunteered for test. How many different treatment groups are possible?

Solution: We have 20 distinct volunteers and we are selecting 15 of them. It does not matter what order the people are selected in. They all are just part of the same treatment group.

The number of combinations of **20 distinct items chosen 15 at a time without regard to order is**

$$_{20}C_{15} = 15{,}504$$

How can I tell if a problem involves a Permutation or a Combination ?

If the problem **specifically asks for the number of permutations** then every different ordering must be counted and the formula for permutations can be used $_{n}P_{r}$. If the problem **specifically asks for the number of combinations** then every different ordering must be counted and the formula for permutations can be used $_{n}P_{r}$

Most problems do not specifically ask for a permutation or a combination. Most problems ask for the number of arraignments and require you to determine which one is correct, You must ask if it makes a difference if an item is selected first rather than second. If the order of selection make a difference then the problem requires a permutation answer. If the order of selection does not make a difference then the problem requires a combination answer.

Example 1

Permutation

Given a group of 10 people, 3 of the 10 people are to be selected and given a prize. The first person selected gets a $ 10 prize, the second person selected gets a $ 5 prize and the third person selected gets a $ 1 prize. The order of selection makes a difference in the prize awarded so the problem requires a **permutation** answer $_{10}P_{3}$

Combination

Given a group of 10 people, 3 of the 10 people are to be selected and given a prize. All three of the prizes are $ 25. The order of selection **DOES NOT** make a difference in the prize awarded so the problem requires a **combination** answer $_{10}C_{3}$

Example 2

Permutation

Given a group of 12 people, 2 of the 12 people are to be selected to serve on a committee. The first person selected will be the President, the second person selected will be the Vice President. The order of selection makes a difference in the office they get so the problem requires a **permutation** answer $_{12}P_{2}$

Combination

Given a group of 12 people, 2 of the 12 people are to be selected to serve on a committees as committee member. The order of selection **DOES NOT** make a difference as they will both be committee members in either case. The problem requires a **combination** answer $_{12}C_{2}$