Permutations
(Order DOES Matter)

A **permutation** is a **distinct** **arrangement of n DIFFERENT ITEMS** with **none of the items being REPEATED** **where different arraignments of the same items are considered separate arraignments.**

1. You have n items to chose from. Each of **the n items are different.**
2. You select r items. Each **different ordering of the same items** is counted as a different **permutation**. If we change the order of 2 items they are different permutations.

The number of **permutations** of **n different items chosen r at a time is**

$$_{n}P_{r}$$

Example

Joe, Bob and Sue are candidates for President and Vice president. The one with the most votes will be the president and the one with the second most votes will be Vice President. We will list the president First and the Vice P resident second. How many different outcomes can the election have?

There are **6 different Arraignments** of the three people **Bob, Sam and Joe taken two names at a time**

Joe (Pres.), Bob (VP) **Sue (Pres.), Joe (VP)** **Bob (Pres.), Sue (VP)**

Bob (Pres.) , Joe (VP) **Joe (Pres.), Sue (VP)** **Sue (Pres.), Bob (VP)**

Each **different ordering** of the same items is counted as a **different Permutation** The order of the items **does mater**.

Joe (Pres.), Bob (VP) and **Bob (Pres.) , Joe (VP)** both contain Joe and Bob **but in a different order so they are counted as 2 different permutations**

Sue (Pres.) , Joe (VP) and **Joe (Pres.), Sue (VP)** both contain Sue and Joe **but in a different order so they are counted as 2 different permutations**

Bob (Pres.) , Sue (VP) and **Sue (Pres.) , Bob (VP)** both contain Sue and Bob **but in a different order so they are counted as 2 different permutations**

there are **6 different Permutations** of the three people **Bob, Sam and Joe taken two names at a time**

The number of **permutations** of **3 items chosen 2 at a time is**

$$_{10}P_{2} = 6$$

Using the Calculator to find the value of ${}_{n}P_{r}$

1. Clear the screen with the **CLEAR** key
2. Press the number keys to display the value of **n**
3. Push the PRB key to get the 3 probability selections ${}_{n}P_{r}$ ${}_{n}C_{r}$!
4. Press the Right Arrow key ▶ to get the underline under the ${}_{n}P_{r}$ symbol.
5. Press the ENTER = key to select the ${}_{n}P_{r}$ symbol.
6. Press the number keys to input the value of **r**
7. A display like $3\ {}_{n}P_{r}\ 2$ will be shown.
8. To find the value of $3\ {}_{n}P_{r}\ 2$ press the ENTER = key.

Example 1

$_{14}P_{5} = 240{,}240$

Example 2

$_{12}P_{4} = 11{,}880$

Example 3

$_{9}P_{5} = 15{,}120$

Example 4

There are **12 different ways** that the **four** numbers **1, 2 3 and 4** can be **arranged**
to make a two digit number
with **none of the digits being repeated.**

1 2	**1 3**	**1 4**	**2 1**	**2 3**	**2 4**
3 1	**3 2**	**3 4**	**4 1**	**4 2**	**4 3**

1. The 4 numbers we started with (1 , 2, 3 and 4) are all distinct (different).
2. None of the digits were repeated in any arrangement. i.e. We did not use 3 3 or 2 2
   This means we selected from 1 , 2 , 3 4 without replacement
3. Each of the 12 **arrangements is different.** If we change the order we get a different number.

Each of the 12 arrangements is a **permutation of**
4 different numbers picking 2 of the numbers without replacement

we call the list of numbers the 12 permutations of 4 items taken **2 at a time**

and use the notation $_{4}P_{2} = 12$

Example 5

There are **6 different ways** that the **three** letters **A, B, C** can be **arranged to make a two letter word** with **none of the letters being repeated.**

A B	**A C**	**B A**	**B C**	**C A**	**C B**

1. The 3 letters we started with are all distinct (different).
2. None of the letters were repeated in an arrangement. i.e. We did not use A A or B B
   This means we selected from A , B, C without replacement
3. Each of the 6 **arrangements is different.** If we change the order we get a different word.

Each of the 6 arrangements is a **permutation of the 3 different letters picking 2 of the letters** without replacement

we call the list the 6 permutations of 3 items taken **2 at a time**

and use the notation

$${}_3P_2 = 6$$

Example 6

In how many different was can **10 students** running for the **3 offices** of President, Vice President and Treasure be elected if no student can hold more than one office?

Solution: We have 10 distinct people and we are selecting 3 of them at a time. No person can hold 2 offices (without replacement) and the order of selection matters.

n = 10 and r = 3

$${}_{10}P_3 = 720$$

Example 7

How may different 2 letter words can be made using the letters A, B C, D E, F , G if no letter can be used more than once.

Solution: We have 7 distinct letters and we are selecting 3 of them at a time. No letter can be used twice (without replacement) and the order of letters make different words so **order matters.**

n = 7 and r = 2

$${}_7P_2 = 42$$

Example 8

7 people volunteered to serve on a three member committee. One will be the President, one the Vice President, and one the Treasurer. No one can serve in more than one position. How may different committees can be formed?

Solution: The first person picked will be President, the second person picked will be Vice President, and the third person picked will be Treasurer. No person can hold two offices (without replacement). The **order** of selection **changes which office is held (order matters)** so this is a permutation problem.

The number of permutations of **7 items chosen 3 at a time is**

$$_{7}P_{3} = 210$$

There are 210 different arraignments of President, Vice President, and Treasurer if 10 different people can be selected to fill the 3 positions.

Example 9

15 people each buy **1 ticket** to a drawing. The first person whose number is selected in the drawing wins $ 100, the second person selected will win $ 50, the third person selected will win $ 25 and the 4th person selected will get $ 10. How may different ways to give out the awards are possible?

Solution: No person can win twice because they each have only 1 ticket. The **order** of selection produces a different prize (**order matters)** so this is a permutation problem.

The number of permutations of **15 items chosen 4 at a time is**

$$_{15}P_{4} = 37{,}760$$

There are 37,760 different ways to give out the awards to 15 ticket holders if there are 4 different prizes to be awarded.

Example 10

How many different 8 letter words (real or imaginary) can be created using all the letters in the word **SCRAMBLE** ?

Solution:

Each of the 8 letters in the word **SCRAMBLE** are different. No letter can be used twice in making the word and a different order of the letters produces a different word (**order matters**) so this is a permutation problem.

The number of permutation of **8 items chosen 8 at a time is**

$$_{8}P_{8} = 42{,}320$$

There are 42,320 different different 8 letter words (real or imaginary) that can be created using all the letters in the word **SCRAMBLE** ?

The permutation problems above required n distinct (different) items but allowed you to select as many of the n items at a time as you wanted. You could use the 10 distinct items to produce a permutation of 5 items. A special case of permutations **exists.** It requires that you **have n items** and use a**ll of the n item**s one time to produce a 10 item permutation but it allows for **some of the n items to be repeated.**

A Permutation Rule for the number of different arraignments of n items
using ALL of the n items one time
when some of the items are repeated

How may different 3 letter arraignments are there using the 3 letters in the word SEE. To help follow the solution we will label the 2 letters E as E_1 and E_2

S E_1 E_2 = **SEE**	E_1 E_2 S = **EES**	E_2 S E_1 = **ESE**
S E_2 E_1 = **SEE**	E_2 E_1 S = **EES**	E_1 S E_2 = **ESE**

It looks like there are 6 different 3 letter words possible using the 3 letters but some of the 6 words are repeats and cannot be counted twice. There are **only 3** different 3 letters arraignment of the letters.

A Permutation Rule for the number of different arraignments of n items
using all of the n items one time
when some of the items are repeated

There are n items to chose from.

There are n_1 of one item and n_2 of a second item and n_3 of a third item and .... n_k of k^th item

1. There are **n items to chose from** and **some of the items are repeated**.
2. **Each of the n items are selected once without replacement**
3. **Different arrangements are** considered **different.**

The total number of permutations of n items using all n items without replacement where one item occurs n_1 times and a second item occurs n_2 times and a third item occurs n_3 times ... etc.

$$= \frac{n!}{n_1! \bullet \; n_2! \bullet n_3! \bullet\bullet \; n_k!}$$

Example 10

How many different ways can 10 flags be arraigned to make a 10 flag display if you must use 3 Red flags, 5 Blue flags and 2 Green flags?

Solution. There are 10 flags and we are making a 10 flag display using each flag one time.

There are a total of 10 flags n = 10

3 Red flags means $n_1 = 3$ 5 Blue flags means $n_2 = 3$ 2 Green flags means $n_3 = 2$

$$\text{The total number of permutations} = \frac{n!}{n_1! \bullet n_2! \bullet n_3! \bullet \bullet \; n_k!} = \frac{10!}{3! \bullet 5! \bullet 2!} = 2520$$

Example 11

How may different 5 letter words are there using all the the letters in the word SEEDS without replacement?

Solution. There are 5 letter and we are making 5 letter words using each of the 5 letter in SEEDS one time.

There are a total of 5 letters n = 5

there are 2 S's $n_1 = 2$ there are 2 E's $n_2 = 2$ there is 1 D = 1

$$\text{The total number of permutations} = \frac{n!}{n_1! \bullet n_2! \bullet n_3! \bullet \bullet \; n_k!} = \frac{5!}{2! \bullet 2! \bullet 1!} = 30$$

