Section 4 – 5: **Counting Techniques**

The formula for finding the probability that an Even E happens is:

$$P(E) = \frac{\text{the number of ways E can happen}}{\text{the total number of outcomes in the sample space}}$$

This formula requires that the total number of ways that Event E can happen is known or can be found. It also requires that the total number of outcome in the sample space is known or can be found.

Finding the total number of outcomes in the sample space

Finding the total number of outcomes in the sample space is the first step in finding the value of P(E). In past examples in this chapter the sample space was given and the total outcomes could be be counted. In some cases a formula was used to find the total number of outcomes in the sample space. In other cases a branching diagram was used to find the total number of outcomes in the sample space. This section will introduce a counting technique that will help in finding the total number of outcomes in the sample space.

Multiplication Rule for a Sequence of Events

A sequence of events occurs and the **first event can happen in a ways** and the **second event can happen in b ways** and the t**hird event can happen in c ways** and so on for other events. The total number of possible outcomes from the sequence of events is

a • b • c.... ways.

Example 1

How many **2 letter words** be made with the letters A, B and C if **no letter can be repeated?**

Solution:

The total number of 2 letter words is = The number of ways the first letter can be chosen • The number of ways the second letter can be chosen

The total number of 2 letter words is = _____ • _____

There are 3 letters (A , B and C) that can selected in the **first event**.

Total = 3 • _____

There are 2 letters left (1 was used and cannot be reused) that can selected in the **second event.**

Total = 3 • 2 = 6

Solution: There are 3 • 2 = 6 **2 letter words** that can be made with the letters A, B and C if **no letter can be repeated?**

Example 2

How many **2 letter words** be made with the letters A, B and C if **the letters can be repeated?**

Solution:

The total number of 2 letter words is = The number of ways the first letter can be chosen • The number of ways the second letter can be chosen

The total number of 2 letter words is = _____ • _____

There are 3 letters (A , B and C) that can selected in the **first event**.

Total = 3 • _____

There are 3 letters (all 3 can be reused) that can selected in the **second event.**

Total = 3 • 3 = 9

Solution: There are 3 • 3 = 9 **2 letter words** that can be made with the letters A, B and C if **the letters can be repeated?**

Example 3

How many **3 digit** numbers be made with the four numbers 6, 7, 8 , and 9 if **no number can be repeated?**

Solution:

The total number of 3 digit numbers is = _____ • _____ • _____

There are 4 numbers (6 , 7, 8 , and 9) that can selected in the **first event**.

Total = 4 • _____ • _____

There are 3 numbers left (1 was used and cannot be reused) that can selected in the **second event.**

Total = 4 • 3 • _____

There are 2 numbers left (2 were used and cannot be reused) that can selected in the **third event.**

Total = 4 • 3 • 2 = 24

Solution: There are 4 • 3 • 2 = 24 **3 digit** numbers be made with the numbers 4, 5, 6 , and 7 if **no number can be repeated?**

Example 4

How many **3 digit** numbers be made with the four numbers 6, 7, 8 , and 9 if **the numbers can be repeated?**

Solution:

The total number of 3 digit numbers is = _____ • _____ • _____

There are 4 numbers (6 , 7, 8 , and 9) that can selected in the **first event**.

Total = 4 • _____ • _____

There are 4 numbers (the numbers can be reused) that can selected in the **second event.**

Total = 4 • 4 • _____

There is 4 numbers (the numbers can be reused) that can selected in the **third event.**

Total = 4 • 4 • 4 = 64

Solution: There are 4 • 4 • 4 = 64 **3 digit** numbers be made with the numbers 4, 5, 6 , and 7 if **the numbers can be repeated?**

Example 5

Set A = { 1 , 2 , 3 , 4 , 5 }

How many **4 digit numbers** can be made with the numbers in set A if **no number can be repeated?**

There are 5 numbers that can be used to fill in the **first blank**.

5 • _____ • _____ • _____

There are 4 numbers left (1 was used and cannot be reused) that can be used to fill in the **second blank.**

5 • 4 • _____ _____

There are 3 numbers left (2 were used and cannot be reused) that can be used to fill in the third blank.

5 • 4 • 3 _____

There are 2 numbers left (3 were used and cannot be reused) that can be used to fill in the forth blank.

5 • 4 • 3 • 2

Solution: There are 5 • 4 • 3 • 2 = 120 4 digit numbers that can be made using the numbers in Set A.

Example 6

Set B = $\{ 1 , 2 , 3 , 4, 5 \}$

How many **3 digit numbers** can you make with the numbers in Set B if **the numbers can be repeated** and the **first two characters** must be an even number ?

There are 2 even numbers that can be used to fill in the first blank (2 or 4).

2 • _____ • _____

The numbers can be reused so there are sill 2 numbers left (2 or 4) that can be used to fill in the second blank .

2 • 2 • _____

There are 5 numbers (the numbers can be reused) that can be used to fill in the third blank .

2 • 2 • 5

Solution: There are 2 • 2 • 5 = 10 3 digit numbers that can be made using the numbers in Set B if **the numbers can be repeated** and the **first two characters** must be an even number.

Example 7

Set C = $\{ 1 , 2 , 3 , 4 , B , C , D \}$

How many **4 character license plates** can be made with the **7 characters** in Set C if **no characters can be repeated** and the **first two characters** must be a number ?

There are 4 numbers that can be used to fill in the first blank.

4 • _____ • _____ • _____

There are 3 numbers left (1 number was used and cannot be reused) that can be used to fill in the second blank.

4 • 3 • _____ _____

There are 5 numbers or letters left (2 numbers were used and cannot be reused) that can be used to fill in the third blank.

4 • 3 • 5 _____

There are 4 numbers or letters left (3 characters were used and cannot be reused) that can be used to fill in the fourth blank.

4 • 3 • 5 • 4

Solution: There are 4 • 3 • 5 • 4 = 240 **4 character license plates** that can be made using the numbers in Set C

Example 8

Set D = $\{ E , P , 2 , 3 , 4, 5 \}$

How many 3 character passwords can be made with the **7 characters** in Set D that start with a **Letter** and end with a **prime number** if **no characters can be repeated?**

There are 2 letters that can be used to fill in the first blank.

2 • _____ • _____

There are 3 prime numbers that can be used to fill in the last blank. You must fill in the blanks that have a given requirement before you fill in the others.

2 • _____ • 3

There are 4 numbers or letters left (1 letter and 1 prime number were used and cannot be reused) that can be used to fill in the middle blank

2 • 4 • 3

Solution: There are 2 • 4 • 3 = 24 3 character passwords that can be made with the **6 characters** in Set D that start with a **Letter**, and end with a **prime number** if **no characters can be repeated?**

Example 9

Set E = $\{ E , A, P , 3 , 4 , 5 \}$

How many five character passwords can be made from the 6 characters in Set D that start with a **letter end with a letter and have a letter in the middle** if **no characters can be repeated?**

There are 3 letters that can be used to fill in the first blank.

3 • _____ • _____ • _____ • _____

There are 2 letters left (1 has been used and cannot be reused) that can be used to fill in the last blank. You must fill in the blanks that have a given requirement before you fill in the others.

3 • _____ • _____ • _____ • 2

There is 1 letter left (2 letters have been used and cannot be reused) that can be used to fill in the middle blank. You must fill in the blanks that have a given requirement before you fill in the others.

3 • _____ • 1 • _____ • 2

There are 3 numbers left (the 3 letters have been used and cannot be reused) that can be used to fill in the second blank.

2 • 3 • 1 • _____ • 3

There are 3 numbers left (1 numbers and 3 letters have been used and cannot be reused) that can be used to fill in the fourth blank.

2 • 3 • 1 • 2 • 3

Solution: There are 2 • 3 • 1 • 2 • 3 = 36 five character passwords can be made from the 6 characters in Set D that start with a **letter, end with a letter and have a letter in the middle** if **no characters can be repeated?**

Example 10

How many different 4 letter words (real or imaginary) can be formed from the 4 letter word **ATOM** if **no letters can be repeated** ?

There are 4 letters that can be used to fill in the first blank.

4 • _____ • _____ • _____

There are 3 letters left (1 letter has been used and cannot be reused) that can be used to fill in the second blank.

4 • 3 • _____ • _____

There are 2 letters left (2 letters have been used and cannot be reused) that can be used to fill in the third blank.

4 • 3 • 2 • _____

There is 1 letter left (3 letters have been used and cannot be reused) that can be used to fill in the forth blank.

4 • 3 • 2 • 1

Solution: There are 4 • 3 • 2 • 1 = 24 different 4 letter words (real or imaginary) that can be formed from the 4 letter word **ATOM** if **no letters can be repeated** ?

Example 11

Set E = $\{ E , A , P , V , W , M \}$

How many 3 **OR** 2 letter words (real ot imaginary) can be formed from the 6 letters in set F if **the letters cannot be repeated?**

_____ • _____ • _____ OR _____ • _____

The number of 3 letter words must be ADDED to the number of 2 letter words to get the total number of 2 or 3 letter words

_____ • _____ • _____ + _____ • _____ • _____

How many **3 letter words**?

There are 6 letters that can be used to fill in the **first blank of the 3 letter words**.

6 • _____ • _____

There are 5 letters left (1 letter has been used and cannot be reused) that can be used to fill in the **second blank of the 3 letter words.**

6 • 5 • _____

There is 4 letters left (2 letters have been used and cannot be reused) that can be used to fill in the **third blank of the 3 letter words.**

6 • 5 • 4 = 120

How many **2 letter words**?

There are 6 letters that can be used to fill in the **first blank of the 2 letter words.**

6 • _____

There are 5 letters left (1 letter has been used and cannot be reused) that can be used to fill in the **second blank of the 2 letter words.**

6 • 5 = 30

Solution: The **number of 3 letter words** must be **ADDED** to the **number of 2 letter words** to get the total number of 2 **OR** 3 letter words. There are 120 possible three letter words. There are 30 possible two letter words for a total of 150 possible two or three letter words.

Example 12

A) How many three letter words exist if the words **cannot have repeated letters** in them?

B) What is the **probability** that I select a three letter word that cannot have repeated letters and it starts with the letter A?

3 letter words

There are 26 letters in the alphabet that can be used to fill in the **first blank of the 3 letter words**.

26 • _____ • _____

There are 25 letters left (1 letter has been used and cannot be reused) that can be used to fill in the s**econd blank of the 3 letter words.**

26 • 25 • _____

There is 24 letters left (2 letters have been used and cannot be reused) that can be used to fill in the **third blank of the 3 letter words.**

26 • 25 • 24 = 15,600

3 letter words that begin with the letter A

There is 1 letter A that can be used to fill in the **first blank of the 3 letter words that begin with the letter A.**

1 • _____ • _____

There are 25 letters left (the letter A has been used and cannot be reused) that can be used to fill in the **second blank of the 3 letter words that begin with the letter A.**

1 • 25 • _____

There is 24 letters left (2 letters have been used and cannot be reused) that can be used to fill in the **third blank of the 3 letter words that begin with the letter A.**

1 • 25 • 24 = 600

Solution: There are 15,600 three letter words exist if the words **cannot have repeated letters** in them. There are 600 three letter words exist if the words **cannot have repeated letters** in them start with the letter A?

The probability of a three letter word starting with the letter A $= \frac{600}{15600} = .04$

