

Section 4 – 3E: Conditional Probabilities A Given B

If we consider all three teams listed in the table below it is clear that there are **45 total players distributed between 3 different teams and 3 different positions**. The Sample Space of all the Guards, Forwards and Centers on all the 3 teams is shown by the 9 frequency cells colored blue. This sample space has 45 players in it.

The Sample Space for all 45 players

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshman Team	4	7	9	20
Total	13	20	12	45

Conditional Notation (A | B)

The Coach calls a practice for **only the centers on all three teams**. He does not require all 45 players on the three teams to show up. He requires only the Varsity Centers, the Jr. Varsity Centers and the Freshmen Centers to attend. There will be a total of 12 centers at the practice.

The Sample Space GIVEN only centers

	Centers
Varsity Team	2
Jr. Varsity Team	1
Freshman Team	9
Total	12

How many players **AT PRACTICE** are Varsity players **GIVEN** that **only Centers** attended

The Conditional Notation we use to ask that questions is written

Varsity | Centers

and is read Varsity **GIVEN** Only Centers

and asks

How many **Varsity Players** are there

GIVEN

only Centers are in the sample space

$$\text{Varsity | Centers} = 2$$

Probability Notation (A | B)

Example 1

$$P(\text{Varsity} | \text{Centers}) =$$

The probability question $P(\text{Varsity} | \text{Centers})$ asks **what is the probability** that if I select one player I will **get a varsity player GIVEN** that **the sample space is only centers**.

The sample space is not the entire 45 players. The **sample space is reduced to the 12 centers**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshman Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only Centers

	Centers
Varsity Team	2
Jr. Varsity Team	1
Freshman Team	9
Total	12

After we **reduce the sample space** to meet the **GIVEN condition** we can ask the following question:

What is the probability that if I select one player I will **get a varsity player GIVEN** that **the sample space is only centers**

$$P(\text{Varsity} | \text{Centers}) =$$

There are 2 Varsity Players **given** we are only looking at 12 Centers.

$$P(\text{Varsity} | \text{Centers}) = \frac{\text{the number of Varsity} | \text{Centers}}{\text{the total number of Centers}} = \frac{2}{12} = .17$$

Example 2

$$P(\text{Guards} \mid \text{Freshmen Team}) =$$

The probability question $P(\text{Guards} \mid \text{Freshmen Team})$ asks **what is the probability** that if I select one player I will **get a Guard** **GIVEN** that **the sample space is only the Freshmen Team**.

The **sample space is reduced to the 20 freshmen players**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshman Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only the Freshmen Team

	Guards	Forwards	Centers	Total
Freshman Team	4	7	9	20

After we reduce the sample space to meet the **GIVEN condition** we can ask the following question:

What is the probability that if I select one player I will **get a Guard**
GIVEN
that **the sample space is only the Freshmen Team**

$$P(\text{Guards} \mid \text{Freshmen Team}) =$$

There are 4 Guards **given** we are only looking at 20 Freshmen Players.

$$P(\text{Guards} \mid \text{Freshmen Team}) = \frac{\text{the number of Guards} \mid \text{Freshmen Team}}{\text{the total number of Freshmen Players}} = \frac{4}{20} = .20$$

Example 3

$$P(\text{Forwards} \mid \text{Jr. Varsity}) =$$

The probability question $P(\text{Forwards} \mid \text{Jr. Varsity})$ asks **what is the probability** that if I select one player I will **get a Forward GIVEN** that **the sample space is only the Jr. Varsity Team**.

The **sample space is reduced to the 15 JV players**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshman Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only the Jr. Varsity Team

	Guards	Forwards	Centers	Total
Jr. Varsity Team	6	8	1	15

After we reduce the sample space to meet the **GIVEN condition** we can ask the following question:

What is the probability that if I select one player I will **get a Forward GIVEN** that **the sample space is only the Jr. varsity Team**

$$P(\text{Forwards} \mid \text{Jr. Varsity}) =$$

There are 8 Forwards **given** we are only looking at 15 JV Players.

$$P(\text{Forwards} \mid \text{Jr. Varsity}) = \frac{\text{the number of Forwards} \mid \text{Jr. Varsity}}{\text{the total number of Jr. Varsity Players}} = \frac{8}{15} = .53$$

Example 4

$$P(\text{Freshman} \mid \text{Forwards}) =$$

The probability question $P(\text{Freshman} \mid \text{Forwards})$ asks **what is the probability** that if I select one player I will **get a Freshman** **GIVEN** that **the sample space is only the Forwards**.

The **sample space is reduced to the 20 Forwards**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only Forwards

	Forwards
Varsity Team	5
Jr. Varsity Team	8
Freshmen Team	7
Total	20

After we reduce the sample space to meet the **GIVEN condition** we can ask the following question:

What is the probability that if I select one player I will **get a Freshman** **GIVEN** that **the sample space is only the Forwards**

$$P(\text{Freshman} \mid \text{Forwards}) =$$

There are 7 Freshman **given** we are only looking at 20 Forwards.

$$P(\text{Freshman} \mid \text{Forwards}) = \frac{\text{the number of Freshman} \mid \text{Forwards}}{\text{the total number of Forwards}} = \frac{7}{20} = .35$$

Example 5

$$P(\overline{\text{Varsity}} \mid \text{Guards}) =$$

The probability question $P(\overline{\text{Varsity}} \mid \text{Guards})$ asks **what is the probability** that if I select one player I will **get a** player that is **NOT** a Varsity Player **GIVEN** that **the sample space is only the Guards**.

The **sample space is reduced to the 13 Guards**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only Guards

	Guards
Varsity Team	3
Jr. Varsity Team	6
Freshmen Team	4
Total	13

There are 10 Not Varsity players **given** we are only looking at the 13 Guards

$$P(\overline{\text{Varsity}} \mid \text{Guards}) = \frac{\text{the number of } \overline{\text{Varsity}} \mid \text{Guards}}{\text{the total number of Guards}} = \frac{10}{13} = .77$$

Example 6

$$P(\overline{\text{Centers}} \mid \text{Varsity}) =$$

The probability question $P(\overline{\text{Varsity}} \mid \text{Guards})$ asks **what is the probability** that if I select one player I will **get a player** that is **NOT** a Center **GIVEN** that **the sample space is only the Varsity**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only **Varsity Players**

The **sample space is reduced to the 10 Varsity Players**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10

There are 8 Not Centers **given** we are only looking at the 10 Varsity Players

$$P(\overline{\text{Centers}} \mid \text{Varsity}) = \frac{\text{the number of } \overline{\text{Centers}} \mid \text{Varsity}}{\text{the total number of Varsity}} = \frac{8}{10} = .80$$

Example 7

$$P(\overline{\text{Guards}} \mid \overline{\text{Varsity}}) =$$

The probability question $P(\overline{\text{Guards}} \mid \overline{\text{Varsity}})$ asks **what is the probability** that if I select one player I will **get** a player that is **NOT** a Guard **GIVEN** that **the sample space is NOT Varsity**.

The **sample space is reduced to the 35 NOT Varsity Players**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only **NOT Varsity Players**

The **sample space is reduced to the 35 NOT Varsity Players**.

	Guards	Forwards	Centers	Total
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20

There are 25 Not Centers **given** we are only looking at the 35 NOT Varsity Players

$$P(\overline{\text{Guards}} \mid \overline{\text{Varsity}}) = \frac{\text{the number of } \overline{\text{Guards}} \mid \overline{\text{Varsity}}}{\text{the total number of } \overline{\text{Varsity}}} = \frac{25}{35} = .71$$

Example 8

$$P((\text{Guards OR Centers}) \mid \overline{\text{Freshman}}) =$$

The probability question $P((\text{Guards OR Centers}) \mid \overline{\text{Freshman}})$ asks **what is the probability** that if I select one player I will **get a** player that is **NOT** a Guard or a Center **GIVEN** that the **sample space is NOT Freshman Players**.

The **sample space is reduced to the 25 NOT Freshman Players**.

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only **NOT Freshman Players**

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15

There are 12 Guards or Centers **given** we are only looking at the 25 NOT Freshman Players

$$P((\text{Guards OR Centers}) \mid \overline{\text{Freshman}}) = \frac{12}{25} = .48$$

Example 9

$$P(\overline{\text{Jr. Varsity}} \mid \overline{\text{Forwards}}) =$$

The **sample space is reduced to the 25 NOT Forwards.**

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only **NOT Forwards**

	Guards	Centers
Varsity Team	3	2
Jr. Varsity Team	6	1
Freshmen Team	4	9
Total	13	12

There are 18 NOT Jr. Varsity Players **given** we are only looking at the 25 NOT Forwards

$$P(\overline{\text{Jr. Varsity}} \mid \overline{\text{Forwards}}) = \frac{18}{25} = .72$$

Example 10

$$P(\overline{\text{Varsity}} \mid \overline{\text{Centers}}) =$$

The **sample space is reduced to the 33 NOT Centers**

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Freshmen Team	4	7	9	20
Total	13	20	12	45

Sample Space **GIVEN** only **NOT Centers**

	Guards	Forwards
Varsity Team	3	5
Jr. Varsity Team	6	8
Freshmen Team	4	7
Total	13	20

There are 25 NOT Varsity **given** we are only looking at the 33 NOT Centers

$$P(\overline{\text{Varsity}} \mid \overline{\text{Centers}}) = \frac{25}{33} = .76$$