Section 4 – 2B: Conditional Probability Involving Independent Events

P (A and B) independent Case

If Event A and Event B are **INDEPENDENT** then
P (A AND B) = P (A) • P (B)

2 marbles with replacement

A bag contains **10 marbles** { 1R , 4 G , 5 B }. You randomly **draw 2 marbles.** You randomly draw one marble from the bag, record it's color, and then **replace the first marble drawn**. You then randomly draw a second marble from the bag and record it's color. **The first marble drawn is replaced in the bag before the second draw.** The bag is unchanged after the first event so the outcome of the first event has no effect on the second draw. The two events are **independent**.

The bag's contents at the start of the **first selection**		The bag's contents at the start of the **second selection**
1 R 4 G 5 B	and then	1 R 4 G 5 B
10 marbles	**with** replacement	10 marbles

Example 1

A bag contains 10 marbles { 1R , 4G , 5B } and you randomly draw 2 marbles, one at a time, **with replacement.**

Find P(**B and G**)

Find the probability that you get a Blue and then a Green marble**.**

Solution:

You randomly draw 2 marbles, one at a time **with replacement**. The two events are **independent.**

P(**B and G**) = P(B) •P(G) $= \frac{\text{5 possible Blue marbles}}{\text{10 total marbles}} \bullet \frac{\text{4 possible Green marbles}}{\text{10 total marbles}}$

P(**B and G**) = P(B) •P(G) $= \frac{5}{10} \bullet \frac{4}{10} = \frac{1}{5} = .20$

Example 2

A bag contains 10 marbles { 1R , 4G , 5B } and you randomly draw 2 marbles, one at a time, **with replacement.**

Find P(**G and G**)
Find the probability that you get 2 Green marbles**.**

The bag's contents at the start of the **first selection**		The bag's contents at the start of the **second selection**
1 R 4 G 5 B	and then	1 R 4 G 5 B
10 marbles	**with** replacement	10 marbles

Find P(**G and G**)

Solution:

You randomly draw 2 marbles, one at a time **with replacement**. The two events are **independent.**

To draw 2 Green marbles, one at a time, you need to select a G and then a G.

P(**G and G**) = P(G) • P(G) = $\frac{\text{4 possible Green marbles}}{\text{10 total marbles}} \bullet \frac{\text{4 possible Green marbles}}{\text{10 total marbles}}$

P(**G and G**) = P(G) • P(G) = $\frac{4}{10} \bullet \frac{4}{10} = \frac{4}{25} = .16$

Example 3

A bag contains 15 marbles { 1R , 3G , 5B, 6Y } and you randomly draw 2 marbles, one at a time, **with replacement.**

Find P(**B and Y**)
Find the probability that you get a Blue and then a Yellow marble.

The bag's contents at the start of the **first selection**		The bag's contents at the start of the **second selection**
1 R		1 R
3 G	and	3 G
5 B	then	5 B
6 Y		6 Y
15 marbles	**with** replacement	15 marbles

Find P(**B and Y**)
Solution:

You randomly draw 2 marbles, one at a time **with replacement**. The two events are **independent.**

$$P(\mathbf{B\ and\ Y}) = P(B)\bullet P(Y) = \frac{\text{5 possible Blue marbles}}{\text{15 total marbles}} \bullet \frac{\text{6 possible Yellow marbles}}{\text{15 total marbles}}$$

$$P(\mathbf{B\ and\ Y}) = P(B)\bullet P(Y) = \frac{5}{15} \bullet \frac{6}{15} = \frac{\cancel{5}^{1}}{\cancel{15}^{3}} \bullet \frac{\cancel{6}^{2}}{\cancel{15}^{5}} = \frac{2}{15} \approx .13$$

Example 4

A bag contains 10 marbles { 1R , 4G , 5B } and you randomly draw 2 marbles, one at a time, **with replacement.**

Find P(**G and Y**)
Find the probability that you get a Green and then a Yellow marble.

The bag's contents at the start of the **first selection**		The bag's contents at the start of the **second selection**
1 R 4 G 5 B	and then	1 R 4 G 5 B
10 marbles	**with** replacement	10 marbles

Find P(**G and Y**)

Solution:

You randomly draw 2 marbles, one at a time **with replacement**. The two events are **independent.**

P(**G and Y**) = P(G)•P(Y) $= \frac{\text{4 possible green marbles}}{\text{10 total marbles}} \bullet \frac{\text{0 possible yellow marbles}}{\text{10 total marbles}}$

P(**G and Y**) = P(G)•P(Y) $= \frac{4}{19} \bullet \frac{0}{10} = 0$

Example 5

A bag contains 15 marbles { 1R , 3G , 5B, 6Y } and you randomly draw 2 marbles, one at a time **with replacement.**

Find P(G and $\overline{Y}$)

Find the probability that you get a **Green** marble then a **Not Yellow** marble.

The bag's contents at the start of the **first selection**		The bag's contents at the start of the **second selection**
1 R		1 R
3 G	and	3 G
5 B		5 B
6 Y	then	6 Y
15 marbles	**with** replacement	15 marbles

Find P(G and $\overline{Y}$)

Solution:

You randomly draw 2 marbles, one at a time **with replacement**. The two events are **independent.**

P(G and $\overline{Y}$) = P(G) • P($\overline{Y}$) $= \dfrac{3 \text{ possible Green marbles}}{15 \text{ total marbles}} \bullet \dfrac{9 \text{ possible Not Yellow marbles}}{15 \text{ total marbles}}$

P(G and $\overline{Y}$) = P(G) • P($\overline{Y}$) $= \dfrac{3}{15} \bullet \dfrac{9}{15} = \dfrac{\cancel{3}^{1}}{\cancel{15}^{5}} \bullet \dfrac{\cancel{9}^{3}}{\cancel{15}^{5}} = \dfrac{3}{25}$ = .12

Example 6

A bag contains 12 marbles { 3R , 2G , 6B, 1Y } and you randomly draw 2 marbles, one at a time, **with replacement.**

$P(\overline{R} \text{ and } \overline{B})$

Find the probability that you get a **NOT RED and** then a **Not Blue** marble.

The bag's contents at the start of the **first selection**		The bag's contents at the start of the **second selection**
3 R 2 G 6 B 1 Y	and then	3 R 2 G 6 B 1 Y
12 marbles	**with** replacement	12 marbles

Find $P(\overline{R} \text{ and } \overline{B})$

Solution:

You randomly draw 2 marbles, one at a time **with replacement**. The two events are **independent.**

$$P(\overline{R} \text{ and } \overline{B}) = P(\overline{R}) \bullet P(\overline{B}) = \frac{\text{9 possible Not Red marbles}}{\text{15 total marbles}} \bullet \frac{\text{6 possible Not Blue marbles}}{\text{15 total marbles}}$$

$$P(\overline{R} \text{ and } \overline{B}) = P(\overline{R}) \bullet P(\overline{B}) = \frac{9}{12} \bullet \frac{6}{12} = \frac{\cancel{9}^{3}}{\cancel{12}^{4}} \bullet \frac{\cancel{6}^{1}}{\cancel{12}^{2}} = \frac{3}{8} \approx .38$$

Example 7

A bag contains 12 marbles { 3R , 2G , 6B, 1Y } and you randomly draw 2 marbles, one at a time, **with replacement.**

$P(B \text{ and } \overline{Y} \text{ and } R)$

Find the probability that you get a **Blue and then a NOT Yellow and** then a **Red** marble.

The bag's contents at the start of the **first selection**		The bag's contents at the start of the **second selection**		The bag's contents at the start of the **third selection**
3 R 2 G 6 B 1 Y	and then	3 R 2 G 6 B 1 Y	and then	3 R 2 G 6 B 1 Y
12 marbles	**with** replacement	12 marbles	**with** replacement	12 marbles

$P(B \text{ and } \overline{Y} \text{ and } R)$

Solution:

You randomly draw 3 marbles, one at a time **with replacement**. The 3 events are **independent.**

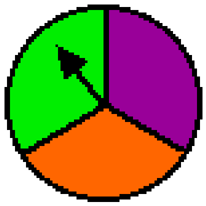
$$P(B \text{ and } \overline{Y} \text{ and } R) = P(B) \bullet P(\overline{Y}) \bullet P(R) = \frac{6}{12} \bullet \frac{11}{12} \bullet \frac{3}{12} = \frac{\cancel{6}^{1}}{\cancel{12}^{2}} \bullet \frac{11}{12} \bullet \frac{\cancel{3}^{1}}{\cancel{12}^{4}} = \frac{11}{96} = .11$$

If Event A and Event B are **INDEPENDENT** then

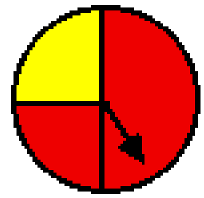
P (A AND B) = P (A) • P (B)

Example 8

Spinner 1



Spinner 2



Find P(**G and R**)
Find the probability that you spin Green **on spinner 1** and then spin **Red on spinner 2**.

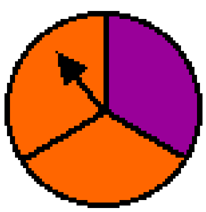
Solution:

The outcome from one spinner does not effect the other so the two events are **independent.**

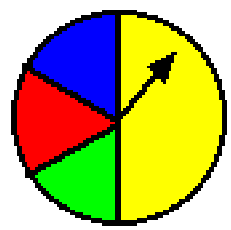
P(**G and R**) = P(G) •P(B) $= \frac{1}{3} \bullet \frac{3}{4} = \frac{1}{4}$

Example 9

Spinner 1



Spinner 2



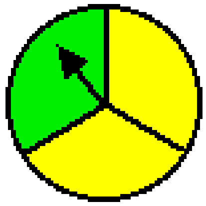
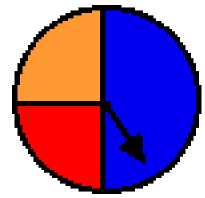
Find the probability that you spin **Tan on spinner 1** and then spin **Red on spinner 2**.

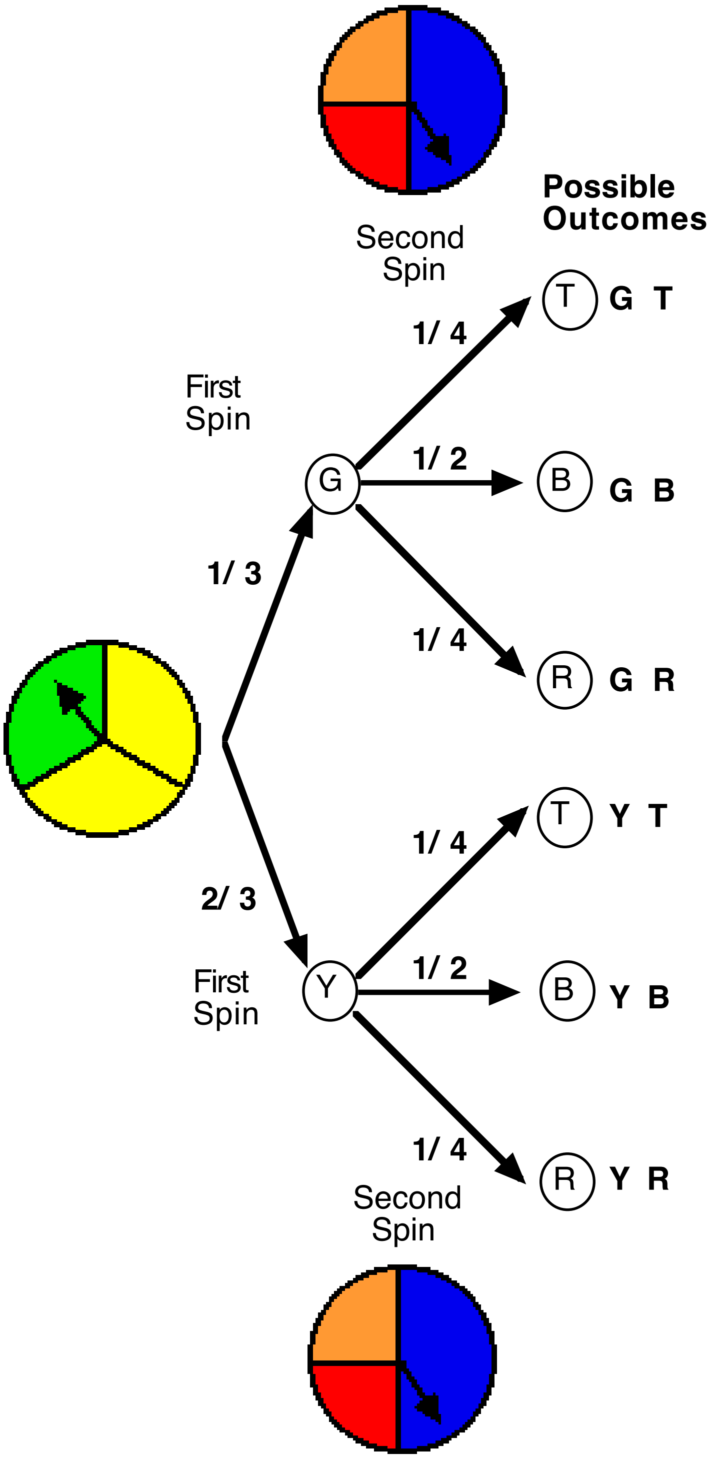
Solution:

The outcome from one spinner does not effect the other so the two events are **independent.**

P(**T and R**) = P(T) •P(R) $= \frac{2}{3} \bullet \frac{1}{6} = \frac{1}{9}$

Example 10

Find all the the probabilities if you spin **spinner 1**  and then spin **Spinner 2** 



Probabilities

P (**G and T**) = $\frac{1}{3} \bullet \frac{1}{4} = \frac{1}{12}$

P (**G and B**) = $\frac{1}{3} \bullet \frac{1}{2} = \frac{1}{6}$

P (**G and R**) = $\frac{1}{3} \bullet \frac{1}{4} = \frac{1}{12}$

P (**Y and T**) = $\frac{2}{3} \bullet \frac{1}{4} = \frac{1}{6}$

P (**Y and B**) = $\frac{2}{3} \bullet \frac{1}{2} = \frac{1}{3}$

P (**Y and R**) = $\frac{2}{3} \bullet \frac{1}{4} = \frac{1}{6}$

Optional Explanation for why P (A AND B) with Independent Events is P (A AND B) = P (A) • P (B)

Procedure: A bag has **1** Red Marble (R) and **2** Blue Marbles (B) in it. You select one marble, **replace it in the bag** and then select a second marble. Find the probability that you will select a **Blue marble first AND** THEN a **Red marble second.**

We write this as **P (Blue AND Red)** but we mean
P (Blue happens first AND then Red happens second)

The sample space of all possible outcomes is shown below.

Second

FIRST	R	B	B
R	R R	R B	R B
B	B R	B B	B B
B	B R	B B	B B

The table to the right shows that 2 of the 9 outcomes have a Blue first and then a Red.

P (Blue AND then Red) = $\frac{2}{9}$

Is there a way to compute the value of this probability **without drawing the sample space?**

P(B) = $\frac{2}{3}$ so 6 of the 9 outcomes will have a Blue first. The probability of the second marble being red is $\frac{1}{3}$. $\frac{1}{3}$ **of the** $\frac{6}{9}$ of the first outcomes that have **Blue first** will have a **Red second**.

P (Blue happens first)

P(B) = $\frac{6}{9}$

FIRST	R	B	B
R	R R	R B	R B
B	B R	B B	B B
B	B R	B B	B B

P (Red happens second)

P(R) = $\frac{1}{3}$

Second

	R	B	B
R	R R	R B	R B
B	B R	B B	B B
B	B R	B B	B B

P (A AND B)

P(B and R) = $\frac{6}{9} \bullet \frac{1}{3} = \frac{2}{9}$

	R	B	B
R	R R	R B	R B
B	B R	B B	B B
B	B R	B B	B B

The yellow cells represent $\frac{1}{3}$ of the $\frac{6}{9}$ of all the possible outcomes.

with Independent Events
P (B AND R) = P(B) •P(R)

P(B and then R) = $\frac{6}{3} \bullet \frac{1}{3} = \frac{2}{9}$

Procedure: Flip a coin and then roll a die. Find the probability that you get a **heads first** and **then roll an even number.**

We write this as **P (H AND Even)** but we mean
P (Heads happens first AND then an Even happens second)

The sample space of possible outcomes from the precede is shown below.

Second

First		**1**	**3**	**5**	**2**	**4**	**6**
	Heads	H 1	H 3	H 5	**H 2**	**H 4**	**H 6**
	Tails	T 1	T 3	T 5	T 2	T 4	T 6

The table to the right shows that 3 of the 12 outcomes have a Head first and then an Even.

P (Head AND then Even) = $\frac{3}{12} = \frac{1}{4}$

Is there a way to compute the value of this probability **without drawing the sample space?**

(H) = $\frac{1}{2}$ so 6 ot the 12 outcomes will have a Head first. The probability of the second number being even is $\frac{1}{2}$. $\frac{1}{2}$ **of the** $\frac{1}{2}$ of the first outcomes that have **Head first** will have an **Even second**.

P (Head happens first)

P(B) = $\frac{1}{2}$

First		**1**	**3**	**5**	**2**	**4**	**6**
	Heads	H 1	H 3	H 5	**H 2**	**H 4**	**H 6**
	Tails	T 1	T 3	T 5	T 2	T 4	T 6

P (Even happens second)

P(R) = $\frac{1}{2}$

Second

	1	**3**	**5**	**2**	**4**	**6**
Heads	H 1	H 3	H 5	**H 2**	**H 4**	**H 6**
Tails	T 1	T 3	T 5	T 2	T 4	T 6

Second

First		**1**	**3**	**5**	**2**	**4**	**6**
	Heads	H 1	H 3	H 5	**H 2**	**H 4**	**H 6**
	Tails	T 1	T 3	T 5	T 2	T 4	T 6

The yellow cells represent $\frac{1}{2}$ of $\frac{1}{2}$ of all the possible outcomes.

with Independent Events
P (H AND then Even) = P(H) •P(E)

P(H and then E) = $\frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$

