Section 4 – 2A: Conditional Probability involving Two Events Introduction

As you board an airplane you notice the plane has 2 engines We will call one **Engine A** and the other **Engine B**. You start to worry about these two engines. How often to they have a part break? How often does a bird fly into an engine and cause it to fail. You start to ask **how likely it is that one of these engines will fail.**

Simple Probability

Simple probability answers the question of the probability that only 1 engine fails.

You wonder what is the probability that Engine A will fail?
We write this as **P (A)**

You may also want to know what is the probability that Engine B will fail?
We write this as **P (B)**

The question we are interested in is "what is the probability of **both engines failing**? There are three ways that this can happen.

Engine A and Engine B fails **at the same time.**

Engine A **fails first** and then Engine B **fails later.**

Engine B **fails first** and then Engine A **fails later.**

Joint Probably

Joint Probably asks the question "What is the probability that Engine A and B both fail **at the exact same time**." This can be stated as **P (A and B fail at the exact same time)**

We write this as **P (A B)**
but is read as P(A and B at the same time)

We **will not cover this probability question** in this chapter. It involves **Bayes Law**. A web search of this topic will produce thousands of pages of material if you are interested.

Conditional Probability

Conditional Probably asks the question "What is the probability that one engine fails first and then the second one fails later. This is the type of probability question we will cover in this chapter.

We write this as **P(A and B)**
but is read as P(A and then B)

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Conditional Probability

Conditional probability is based on the condition that **the first event A has already happened** and now we are concerned about the probability of a second event B happening **GIVEN** that the first event A has already happened.

If the plane is going to crash both engines must fail. Part way to your destination Engine A fails. The probability of the first engine failing is just a simple probability form the least section. When Engine A fails you are now on a **one engine plane**. With Engine A gone you turn your attention to Engine B, the **ONE** remaining engine. You want to know:

What is the probability that Engine B will fail **GIVEN** that Engine A has already failed

We write this question as **P (B | A)**

we read **P (B | A)** as the probability that **B happens given that A has already happened.**

We could also ask

What is the probability that Engine A will fail **GIVEN** that Engine B has already failed

We write this question as **P (A | B)**

we read **P (A | B)** as the probability that **A happens given that B has already happened.**

What is the Difference between **P (A | B) and P (B | A)**

P (A | B) asks what is the probability Engine A will fail **GIVEN** that Engine B **has already failed**

P (B | A) asks what is the probability Engine B will fail **GIVEN** that Engine A **has already failed**

NOTE: In conditional probability, Event A and B **do not occur at the same time.** First one event occurs and then the second one occurs after the first event has already happened. The difference above is in the order the two events occur in.

The values for **P (A | B) or P (B | A)** cannot be found before the concepts of **dependent** and **independent events** are introduced.

The notation P (B |A) was used in the examples above. The next lecture sections will develop the techniques for finding P (B |A) based on drawing marbles from a bag. These problems will help students better understand the concepts dependent or independent events and their relation to finding. P (B |A)

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Finding P (A AND B) If Event A and Event B are **INDEPENDENT**

If the failure of Engine A **does not** effect on the probability that engine B will fail, then event A and event B event are **INDEPENDENT EVENTS**. If Event A and Event B are **INDEPENDENT** then the probability of Engine A AND then Engine B failing is shown below:

If Event A and Event B are **INDEPENDENT** then **P (A AND B) = P (A) • P (B)**

Example 1 (Independent Events):

A pilot states that the probability of Engine A failing on his plane is .05 and that the probability of Engine B (which is a bit older) failing on his plane is .08. The pilot also says that one engine failing **does not effect** the probability of the other engine failing. Find the probability that Engine A fails and then Engine B fails.

Solution: The probability of Engine B failing is Independent on Engine A failing. The two events are independent.

The probability of an engine failure **P (A) =** .05
The probability of an engine failure **P (B) =** .08

Find the probability that Engine A fails and then Engine B fails.

P (A AND B) = P (A) • P (B)

= .05 • .08 = .004

DEPENDENT Events

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If the failure of Engine A **does** effect the probability that Engine B will fail, then we say that the probability of Engine B failing is **dependent** on the failure of Engine A. This may occur because Engine B takes on more stress when Engine A fails. With the engine on a wing instead of the nose the engine must be run at a higher spend to keep the plane flying straight thus increasing the chance of a failure for Engine B.

If the probability of Event B happening **is effected** by the probability of Event A happening then **Event A** and **Event B** are **DEPENDENT**.

If Event A and Event B are **DEPENDENT** then
P (A AND B) = P (A) • P (B I A)

Example **2 (Dependent Events):**

A pilot states that the probability of Engine A failing on his plane is .05. He also says that if **Engine A does fail** then probability of Engine B failing next is .15.

Find the probability that Engine A fails and then Engine B fails.

Solution: The probability of Engine B failing is dependent on Engine A failing.
The two events are dependent.

The probability of an Engine A failing as **P (A) =** .05
The probability of Engine B failing **GIVEN** Engine A has failed is **P (B I A)** = .15

P (A AND B) = P (A) • P (B I A)

P A fails and then B fails) = .05 • .15 = .0075

The notation P (B IA) was used in the examples above. The next lecture sections will develop the techniques for finding P (B IA) based on drawing marbles from a bag. This is a very common problem in basic statistics classes. These problems will help students better understand the concepts dependent or independent events and their relation to finding. P (B IA)

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