Basic Probability Theory

Probability Notation

Every probability problem begins with a **Procedure** that produces a **Sample Space.**

Procedure: You roll one die and record the number shown.

Sample Space: { 1, 2 , 3 , 4 , 5 , 6 }

We now define an event E which is a **subset** of the **outcomes from the sample space.**

event E = { **rolling one even number** }

Given an event E we now ask the question **what is the Probability that the outcome (or outcomes) in event E may happen** if the procedure is performed **a large number of times.** We write this question as **P(event E) or P(E).** The outcomes of event E are listed or described inside the brackets in place of the the letter E.

Example 1

Procedure: You roll one die and record the number shown.

Sample Space: { 1, 2 , 3 , 4 , 5 , 6 }

A) Find P(1) means find the probability of **the outcome being a 1** if the procedure is performed **a large number of times.**

B) Find P(x > 4) means find the probability of **the outcome being a 5 or 6** if the procedure is performed **a large number of times.**

C) Find P(Odd) means find the probability of **the outcome being a 1, 3 or 5** if the procedure is performed **a large number of times.**

Example 2

Procedure: You **select 1 marble** from a bag that contains **2 Blue marbles (B)** and **3 Red marbles (R**)

Sample Space: { **B B R R R** }

A) Find P(B) which means find the probability of **the outcome being 1 Blue marble** if the procedure is performed **a large number of times.**

B) Find P(R) which means find the probability of **the outcome being 1 red marble** if the procedure is performed **a large number of times.**

Finding the Probability of P(E)

If a **sample space has n equally likely outcomes** and an event **E is a subset of the sample space with x outcomes** then

$$P(E) = \frac{x}{n}$$

or in English

$$P(\text{an outcome in event E happens}) = \frac{\text{the number of outcomes in event E}}{\text{the total number of equaly likely outcomes in the sample space}}$$

Examples

Procedure: You roll one die and record the number shown.

Sample Space: { 1, 2 , 3 , 4 , 5 , 6 }

Find **P (4)**

It is **equally likely** that any one of the six numbers { 1, 2 , 3 , **4** , 5 , 6 } is rolled on any single roll. **P(4) means** find the probability of **rolling a 4.** The number 4 is **one of the six numbers in the sample space**. If all six of the outcomes are **equally likely** then it makes sense to say that if we perform the experiment **a large number of times** the probability of **rolling the number 4 should be 1 out of 6.**

$$P(4) = \frac{1}{6}$$

Find **P(x > 4)**

It is **equally likely** that any one of the six numbers { 1, 2 , 3 , 4 , **5 , 6** } is rolled on any single roll. **P(x > 4) means** find the probability of **rolling a 5 or 6.** The numbers **5 and 6** are **two of the six numbers in the sample space**. If all six of the outcomes are **equally likely** then it makes sense to say that if we perform the experiment **a large number of times** the probability of **rolling a 5 or a 6 should be 2 out of 6**

$$P(x > 4) = \frac{2}{6}$$

Find **P(an even number)**

It is **equally likely** that any one of the six numbers { 1, **2** , 3 , **4** , 5 , **6** } is rolled on any single roll. **P(an even number) means** find the probability of **rolling a 2, 4 or 6.** The numbers **2, 4 or 6** are **three of the six numbers in the sample space**. If all six of the outcomes are **equally likely** then it makes sense to say that if we perform the experiment **a large number of times** the probability of **rolling an even number should be 3 out of 6.**

$$P(\text{an even nuber}) = \frac{3}{6}$$

Finding Probabilities P(E)

Example 1

Procedure: You roll one die and record the number shown.

Sample Space: { 1, 2 , 3 , 4 , 5 , 6 }

A) Find P(one odd) which means find the probability of **rolling an odd number.**

Sample Space: { **1** , 2 , **3** , 4 , **5** , 6 }

$$P(\text{rolling one odd number}) = \frac{\text{the number of ways that 1 odd number can be rolled}}{\text{the total number of outcomes in the sample space}} = \frac{3}{6} = \frac{1}{2}$$

B) Find **P(not an odd)** which means find the probability of **rolling a 2, 4, or 6**

Sample Space: { 1, **2** , 3 , **4** , 5 , **6** }

$$P(\text{no odd numbers}) = \frac{\text{the number of ways an odd number cannot be rolled}}{\text{the total number of outcomes in the sample space}} = \frac{3}{6} = \frac{1}{2}$$

C) Find P(x > 1) which means find the probability of **rolling a number greater than 1**

Sample Space: { 1, **2, 3, 4, 5, 6** }

$$P(x > 1) = \frac{\text{the number of numbers greater than 1}}{\text{the total number of outcomes in the sample space}} = \frac{5}{6}$$

D) Find P(x ≤ 4) which means find the probability of **rolling a number less than or equal to 4**

Sample Space: { **1, 2, 3, 4**, 5, 6 }

$$P(x \le 4) = \frac{\text{the number of numbers less than than or equal to 4}}{\text{the total number of outcomes in the sample space}} = \frac{4}{6} = \frac{2}{3}$$

Example 2

Procedure: You catch 1 fish from a farmers stock pond that contains the following fish:
3 trout (T) , 12 bass (B) and 5 Pike (P).

A) Find P(B) which means find the probability of **getting one bass**

Sample Space: { **T T T B B B B B B B B B B B P P P P P** }

$$P(B) = P(\text{getting 1 bass}) = \frac{\text{the number of ways that 1 bass can be selected}}{\text{the total number of outcoms in the sample space}} = \frac{12}{20} = \frac{3}{5}$$

B) Find $P(\overline{B})$ which means find the probability of **getting NO bass**

Sample Space: { **T T T B B B B B B B B B B B P P P P P** }

$$P(\overline{B}) = P(\text{not getting 1 bass}) = \frac{\text{the number of ways that you can select a fish that is not a bass}}{\text{the total number of outcomes in the sample space}} = \frac{8}{20} = \frac{2}{5}$$

C) Find $P(T)$ which means find the probability of **getting one Trout**

Sample Space: { **T T T** B B B B B B B B B B B P P P P P }

$$P(T) = P(\text{getting 1 trout}) = \frac{\text{the number of ways that 1 trout can be selected}}{\text{the total number of outcomes in the sample space}} = \frac{3}{20}$$

D) Find $P(P)$ which means find the probability of **getting one Pike**

Sample Space: { T T T B B B B B B B B B B B **P P P P P** }

$$P(P) = P(\text{getting 1 pike}) = \frac{\text{the number of ways that 1 pike can be selected}}{\text{the total number of outcomes in the sample space}} = \frac{5}{20} = \frac{1}{4}$$

