Basic Probability Theory

Introduction

The study of statistics can be divided into two general areas: **Descriptive Statistics** and **Inferential Statistics**. **Descriptive Statistics** is the act of **collecting, organizing, displaying and summarizing information.** Chapters 1 to 3 developed the concepts involving Descriptive Statistics.

Chapters 5 to 9 will develop the concepts involved with inferential statistics. **Inferential statistics** allows us to take numerical properties about a **sample** of a population and use those properties to **INFER what numerical properties MIGHT be true** about the **entire population.** Because our inference about the population is based on a sample we can **never be 100% certain** that the inference about the population is correct. We will always have some level of uncertainly about our population inferences. **Uncertainty is a normal part of inferential statistics.**

The study of uncertainty is called **Probability Theory**. Probability Theory forms the basis for the methods used in inferential statistics. Because of this, we need to gain a working knowledge of the basics of probability theory before the concepts involving inferential statistics are developed. This chapter acts as a bridge between descriptive statistics and inferential statistics. It will provide the foundation required for the remaining chapters.

Probability Introduction

Let's say that we are going to to pick **one marble** out of a bag of marbles that contains **1 red marble and 9 blue marbles.** We know for certain that we will get a **red or a blue marble** but cannot know which one will be selected. It is easy to see that the outcome from the single pick is not certain. If we put the marble we just picked back in the bag and did the picking again the second outcome could be different from the first outcome.

We cannot make a statement with any certainty about the outcome from picking the marble a single time. Is there anything we can say that is certain about the outcome? **It is certain** that we will get a **Red or a Blue marble.** We can also say that it is certain that we will **NOT get a green marble**.

Even thought **we are uncertain about the outcome** we can say something about the **probable outcome.** **There are nine times more blue marbles than red marbles.** We can say that it is **more probable that we will get a blue marble than a red marble.** We may also say that we should get a blue marble about nine in ten times. We may say that it is **less probable that we will get a red marble than a blue marble.** We may also say that we should get a red marble about one in ten times.

Probability Theory provides **a measure of the likelihood that a specific outcome will occur.** The outcome is uncertain. Probability Theory provides a measure of **the proportion of the total outcomes** a specific outcome will occur if the action is performed **a large number of times.**

Basic Terms

Procedure: A **procedure** is an **action** that has **specific POSSIBLE outcomes**.

Outcome: The results obtained when you do the procedure **one time. The outcome may vary each time you perform the procedure.**

Example of a Procedure and the Possible Outcomes

If the procedure is to **pick one marble** out of a bag of marbles that contains **4 red marbles and 6 blue marbles,** the **specific possible outcomes** could be getting 1 blue marble **or** getting 1 red marble. **Each time we perform the procedure the exact outcome is uncertain.** The outcome may vary each time we perform the procedure.

Sample Space

A procedure has specific **possible** outcomes. We may be uncertain of the exact outcome each time the procedure is performed but we can make a list of **all the possible outcomes**. A list of **all the possible outcomes** if we perform a procedure is called a **Sample Space.**

Sample Space: A list (or picture) of **all the possible outcomes** from performing a procedure.

Examples:

A) If the procedure is to **roll a die** and then **record the number on top** then the sample space of **all the possible outcomes** is shown below.

Sample Space: { 1 , 2 , 3 , 4 , 5, 6 }

B) If the procedure is to **flip a coin two times,** then the Sample Space of all the possible outcomes of the first and second flip are shown below.

Sample Space: { A Head and then a Head (**H H**), A Head and then a Tail (**H T**)
A Tail and then a Head (**T H**), A Tail and then a Tail (**T T**)

or

Sample Space: { HH, HT, TH, TT }

Using a “Branching Diagram” to determine the sample space of 2 or more equally likely events

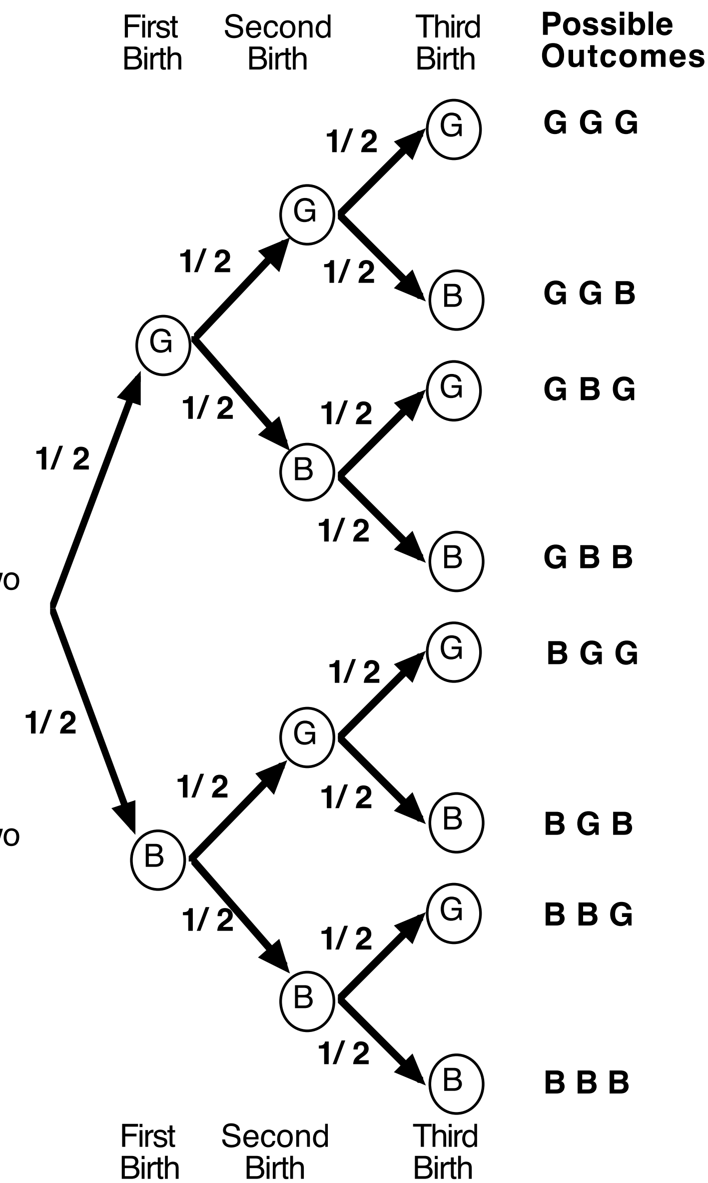
Example 1

C) The procedure is to **have three consecutive single births and record the gender of the babies in birth order**. List the Sample Space of all possible outcomes.

Step 1: List the two possible outcomes for the first birth G and B

Step 2: From each of the G and B’s of the first birth draw two lines that end in one of the two possible outcomes for the second birth G or B

Step 3: From each of the G and B’s of the second birth draw two lines that end in one of the two possible outcomes for the third birth G or B



Sample Space: { **GGG , GGB , GBG , GBB, BGG , BGB , BBG, BBB** }

Note: The chance of getting a Boy or a Girl in any one birth is considered to be equal for this course. The chance of getting a boy is 1/ 2 and the chance of getting a girl is 1/ 2 for each birth Each branch **has an equal chance of occurring.**

Event

A Event is a list of **one or more outcomes from the sample space.** The list may consist of one outcome or several outcomes depending on the description of the event.

An Event may be described by **a list of the specific outcomes**, an **English description** of the outcomes **or an algebraic expression** that describes the outcomes.

Event E

Event: One or more outcomes from the sample space.

Examples:

A) The procedure is to **roll a die one time** and **record the number**

The **Sample Space is** { 1, 2, 3, 4, 5, 6 }
and a **possible Event E could be { an even number} or** { 2, 4, 6 }

B) The procedure is to **randomly select a number from 1 to 10** and **record the number**

The **Sample Space is** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }
and a **possible Event E could be { x > 7} or** { 8, 9, 10 }

$\overline{E}$ The Complement of Event E

The complement of an event E consists of **all the outcomes in the sample space that are not included in event E.** We write the complement of E as $\overline{E}$. We read $\overline{E}$ as **" not E "**

Finding $\overline{E}$ given E and the Sample Space

Examples:

A) If event E is { 2, 3 } and the sample space is { 1, **2, 3**, 4, 5, 6 } a then $\overline{E}$ is everything in the sample space { 1, **2, 3**, 4, 5, 6 } that is not in E.

$$\overline{E} = \{ 1,4, 5, 6 \}$$

B) If event E is { $x \leq 3$ } and the sample space is { **2, 3,** 4, 5, 6 } then $\overline{E}$ is everything in the sample space { **2, 3, 4, 5,** 6, 7 } that is not in E.

$$\overline{E} = \{ x > 3 \} = \{ 4 , 5, 6 \}$$

C) If event E is { numbers less than 10} and the sample space is { **6, 7, 8, 9**, 10, 11 } then $\overline{E}$ is everything in the sample space that is not in E.

$\overline{E}$ = { The number is **10 or more**} = { 10, 11 }

