Section 3 – 3A: Measures of Relative Standing: Z Scores

Relative Position versus Absolute Position

Comparing the distance between two numbers in a data set.

If you were asked how far apart 8 and 20 are you would reply 12 without giving it much thought. You might ask what distance units were involved. Since 8 inches and 20 inches are 12 inches apart and 8 feet and 20 feet are 12 feet apart you may feel better about your answer of 12 if it had a unit of measure attached.

The next question we could ask is **how many standard deviations apart are 8 and 20**. If the 8 and 20 come from a set with a **standard deviation of 2** then 8 and 20 are **6 SD apart.** If the 8 and 20 come from a set with a **standard deviation of 6** then they are only **2 SD apart.**

The distance from 8 to 20 is 12 in absolute distance but the number of standard deviations 8 and 20 are apart is **relative to the size of the standard deviation.** As the standard deviation changes the answer to how many standard deviations 2 numbers are apart changes. If the standard deviation is the scale we use to decide how far apart 2 numbers are apart then the answer will be relative to what the size of the standard deviation is. The use of a scale like the standard deviation that changes with each data means that the answer to how many standard deviations apart 2 numbers are is **relative** to the size of the standard deviation.

In this course, it is very common to ask

"how many standard deviations from the mean a given data point is."

Z Scores

Every data point x in a set of bell shaped data has a z score. The **z score** for any given data point x is defined to be **the number of standard deviations** that a data point **x is away from the mean. in a normal distribution.**

A z score is the number of standard deviations
a data point x is away from the mean of a bell shaped set of data
It is found using the following formula.

$$z = \frac{(x - \text{mean})}{\text{standrad deviation}}$$

Positive Z scores

If the value of **x is larger than the mean** $\bar{x}$ then the value of $(x - \text{mean})$ will be positive and x will have a **Positive z score.**

x values to the **RIGHT of the mean** have **Positive z scores.**

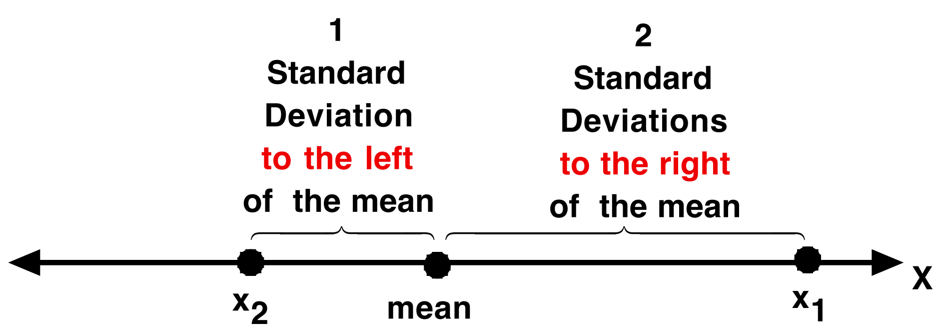
If x has a z score of **positive 2.3** then x is 2.3 standard deviations to the **RIGHT** of the mean.

Negative Z scores

If the value of **x is less than the mean** $\bar{x}$ then the value of $(x - \text{mean})$ will be negative and x will have a **Negative z score.**

x values to the **LEFT of the mean** it will have **Negative z scores.**

If x has a z score of **negative 1.5** then x is 1.5 standard deviations to the **LEFT** of the mean.



x_1 has a Z score of + 2 because x_1 is 2 Standard Deviations to the right of the mean.

x_2 has a Z score of – 1 because x_2 is 1 Standard Deviation to the left of the mean.

Z Scores

A z score is the number of standard deviations
a data point x is away from the mean of a bell shaped set of data

It is found using the one of the following formulas.

Z score for Sample data

$$z = \frac{(x - \bar{x})}{s_x}$$

Z score for Population data

$$z = \frac{(x - \mu_x)}{\sigma_x}$$

Example 1

Converting a x value from a Sample into it's z score

A bell shaped set of data has a **sample mean of** $\bar{x} = 36.8$ and a **sample standard deviation of** $s_x = 4.7$ **Show the set up** and then convert each of the listed data points x from the population into its corresponding z score. **Round to 2 decimal places.**

1A) x = 28.3 z = ________

Set up: $z = \frac{(28.3 - 36.8)}{4.7} = -1.81$

Interpretation:
the data point 28.3
is **negative** 1.81 SD
to the **left of the mean**

1B) x = 48.9 z = ________

Set up: $z = \frac{(48.9 - 36.8)}{4.7} = 2.57$

Interpretation:
the data point 48.9
is **positive** 2.57 SD
to the **right of the mean**

Example 2

Converting a x value from a population into it's z score

A bell shaped set of data has a **population mean of** $\mu = 45.7$ and a **population standard deviation of** $\sigma = 6.2$ **Show the set up** and then convert each of the listed data points x from the population into its corresponding z score. **Round to 2 decimal places.**

2A) x = 38.1 z = ________

Set up: $z = \frac{(38.1 - 45.7)}{6.2} = -1.23$

Interpretation:
the data point 38.1
is **negative 1.23** SD
to the **left of the mean**

2B) x = 56.7 z = .________

Set up: $z = \frac{(56.7 - 45.7)}{6.2} = 1.77$

Interpretation:
the data point 56.7
is **positive** .1.77 SD
to the **right of the mean**

Example 3

Unusual Z scores

A bell shaped set of data has a **population mean of** $\mu = 15.2$ and a **population standard deviation of** $\sigma = 2.1$ **Show the set up** and then convert each of the listed data points x from the population into its corresponding z score. **Round to 2 decimal places.**

2A) x = 10.1 z = ________

Set up: $z = \frac{(10.1 - 15.2)}{2.1} = -2.43$

Interpretation:
the data point 38.1
is **negative 2.33** SD
to the **left of the mean**

the data point 10.1 is more than 2 standard deviation below the mean so it is unusually small for the data

2B) x = 20.3 z = .________

Set up: $z = \frac{(20.3 - 15.2)}{2.1} = 2.42$

Interpretation:
the data point 20.3
is **positive** 2.42 SD
to the **right of the mean**

the data point 20.3 is more than 2 standard deviation above the mean so it is unusually large for the data

