Section 5 – 6B: Solving Logarithmic Equations

Converting a Log Equation with One Log Expression On Each Side of the Equation into an Polynomial Equation

$$\log_b x = \log_b y \quad \text{is equivalent to} \quad x = y$$

Solving Logarithmic Equations with One Log Expression On Each Side of the Equation by converting it into an Polynomial Equation

Example 1

Solve for x:

$\log_2(4x+1) = \log_2(x+7)$

use: $\log_b(\text{x}) = \log_b(\text{x}) \quad \Rightarrow \quad \text{x} = \text{y}$
(there can only be ONE log expression
on each side of the equation to use the rule)

$4x+1 = x+7$

$3x = 6$

$x = 2$

Check: $x = 2$
$\log_2[4(2)+1] = \log_2[2+7]$
$\log_2(9) = \log_2(9)$ Yes

Solution: $x = 2$

Example 2

Solve for x:

$\log_5(2x-5) = \log_5(-x+13)$

use: $\log_b(\text{x}) = \log_b(\text{x}) \quad \Rightarrow \quad \text{x} = \text{y}$
(there can only be ONE log expression
on each side of the equation to use the rule)

$2x-5 = -x+13$

$3x = 18$

$x = 6$

Check: $x = 6$
$\log_5[2(6)-5] = \log_5[-(6)+13]$
$\log_2(7) = \log_2(7)$ Yes

Solution: $x = 6$

Example 3

Solve for x:

$\log_2(2x+1) = \log_2(x^2 - 5x + 7)$

use: $\log_b(\mathrm{x}) = \log_b(\mathrm{x}) \quad \Rightarrow \quad \mathrm{x} = \mathrm{y}$
(there can only be ONE log expression
on each side of the equation to use the rule)

$2x + 1 = x^2 - 5x + 7$

$0 = x^2 - 7x + 6$

$0 = (x-6)(x-1)$

$x = 6$ or $x = 1$

Check: $x = 6$

$\log_2(2x+1) = \log_2(x^2 - 5x + 7)$

$\log_2[2(6)+1] = \log_2[(6)^2 - 5(6) + 7]$

$\log_2(12+1) = \log_2(36 - 30 + 7)$

$\log_2(13) = \log_2(13)$ Yes

Check: $x = 1$

$\log_2(2x+1) = \log_2(x^2 - 5x + 7)$

$\log_2[2(1)+1] = \log_2[(1)^2 - 5(1) + 7]$

$\log_2(2+1) = \log_2(1 - 5 + 7)$

$\log_2(3) = \log_2(3)$ Yes

Solution(s): $x = 6$ or $x = 1$

Example 4

Solve for x:

$\log_5(x) + \log_5(x-4) = \log_5(\text{x}+14)$ use: $\log_b(x) + \log_b(y) = \log_b(x \bullet y)$

$\log_5(x^2 - 4x) = \log_5(x+14)$

use: $\log_b(\text{x}) = \log_b(\text{x}) \Rightarrow \text{x} = \text{y}$
(there can only be ONE log expression
on each side of the equation to use the rule)

$x^2 - 4x = x + 14$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = 7$ or $x = -2$

Check: $x = 7$
$\log_5(x) + \log_5(x-4) = \log_5(\text{x}+14)$
$\log_5(7) + \log_5(7-4) = \log_5(7+14)$
$\log_5(7) + \log_5(3) = \log_5(21)$
$\log_5(7 \bullet 3) = \log_5(21)$ Yes

Check: $x = -2$
$\log_5(x) + \log_5(x-4) = \log_5(\text{x}+14)$
$\log_5(-2) + \log_5(-2-4) = \log_5(-2+14)$
Stop: log (negitive number)
$x \neq -2$

Solution(s): $x = 7$

Example 5

Solve for x:

$\log_7(5x+2)-\log_7(x-2)=2\log_7 3$ use: $\log_b(x)^a = a\bullet\log_b(x)$

$\log_7(5x)-\log_7(x-2)=\log_7 3^2$ use: $\log_b(x)-\log_b(y)= \log_b\left(\frac{x}{y}\right)$

$\log_7\left(\frac{5x+2}{x-2}\right)=\log_7(9)$

use: $\log_b(\mathrm{x})=\log_b(\mathrm{x}) \Rightarrow \mathrm{x}=\mathrm{y}$
(there can only be ONE log expression
on each side of the equation to use the rule)

$\frac{5x+2}{x-2}=9$ note: multiply both sides by $x-2$

$5x-2=9x-18$

$20=4x$

$5=x$

Check: $x=5$
$\log_7\left[5(5)+2\right]-\log_7\left[5-2\right]=2\log_7 3$

$\log_7(25+2)-\log_7(3)=\log_7 3^2$

$\log_7(27)-\log_7(3)=\log_7(9)$

$\log_7\left(\frac{27}{3}\right)=\log_7(21)$

$\log_7(9)=\log_7(9)$ Yes

Solution: $x=5$