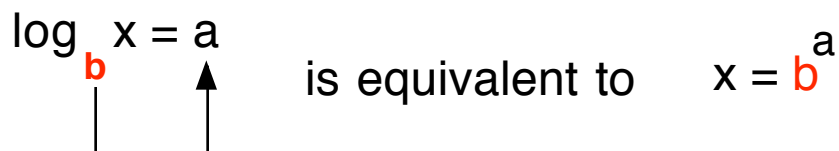


## Section 5 – 6A: Solving Logarithmic Equations

Converting a Log Equation with **ONE Log Expression** into an Exponential Equation

$$\log_b x = a \quad \text{is equivalent to} \quad x = b^a$$


**Solving Logarithmic Equations with ONE log expression in the equation by converting a Log Equation with ONE log expression into an Exponential Equation**

### Example 1

Solve for  $x$ :  $\log_4(5x - 4) = 2$

use:  $\log_b x = a \Rightarrow x = b^a$   
(the base of the log becomes the base under  $a$  as an exponent to convert the log equation into an exponential equation)

$$5x - 4 = 4^2$$

$$5x - 4 = 16$$

$$5x = 20$$

$$x = 4$$

Check:  $x = 4$

$$\log_4[5(4) - 4] = 2$$

$$\log_4(20 - 4) = 2$$

$$\log_4(16) = 2$$

$$16 = 4^2$$

Solution:  $x = 4$

### Example 2

Solve for  $x$ :  $\log_2(4x + 12) = 3$

use:  $\log_b x = a \Rightarrow x = b^a$   
(the base of the log becomes the base under  $a$  as an exponent to convert the log equation into an exponential equation)

$$4x + 12 = 2^3$$

$$4x + 12 = 8$$

$$4x = -4$$

$$x = -1$$

Check:  $x = -1$

$$\log_2[4(-1) + 12] = 3$$

$$\log_2(-4 + 12) = 3$$

$$\log_2(8) = 3$$

$$8 = 2^3 \quad \text{Yes}$$

Solution:  $x = -1$

### Example 3

Solve for x:  $\frac{1}{2}\log_3(7x-5) = 1$

use:  $\log_b x = \log_b(x)^a$

$$\log_3 \sqrt{7x-5} = 1$$

use:  $\log_b x = a \Rightarrow x = b^a$   
(the base of the log becomes the base under a as an exponent to convert the log equation into an exponential equation)

$$\sqrt{7x-5} = 3^1$$

$$\sqrt{7x-5} = 3 \quad \text{Note: square both sides}$$

$$7x - 5 = 9$$

$$7x = 14$$

$$x = 2$$

Check:  $x = 2$

$$\frac{1}{2}\log_3[7(2)-5] = 1$$

$$\frac{1}{2}\log_3(9) = 1$$

$$\log_3 \sqrt{9} = 1$$

$$\log_3 3 = 1$$

$$3 = 3^1 \quad \text{Yes}$$

Solution:  $x = 2$

### Example 3A

Solve for x:  $\frac{1}{2}\log_3(7x-5) = 1$

multiply both sides by 2

$$\log_3(7x-5) = 2$$

use:  $\log_b x = a \Rightarrow x = b^a$   
(the base of the log becomes the base under a as an exponent to convert the log equation into an exponential equation)

$$7x - 5 = 3^2$$

$$7x - 5 = 9$$

$$7x = 14$$

$$x = 2$$

Check:  $x = 2$

$$\frac{1}{2}\log_3[7(2)-5] = 1$$

$$\frac{1}{2}\log_3(9) = 1$$

$$\log_3 \sqrt{9} = 1$$

$$\log_3 3 = 1$$

$$3 = 3^1 \quad \text{Yes}$$

Solution:  $x = 2$

### Example 4

Solve for x:

$$\log(x^2 - x - 19) = 0 \quad \text{Note: Base 10}$$

$$\log_{10}(x^2 - x - 19) = 0 \quad \text{use: } \log_b x = a \Rightarrow x = b^a$$

$$x^2 - x - 19 = 10^0$$

$$x^2 - x - 19 = 1$$

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$x = 5 \text{ or } x = -4$$

$$\text{Check: } x = 5$$

$$\log_{10}[(5)^2 - (5) - 19] = 0$$

$$\log_{10}(25 - 5 - 19) = 0$$

$$\log_{10}(25 - 5 - 19) = 0$$

$$\log_{10}(1) = 0$$

$$1 = 10^0 \quad \text{Yes}$$

$$\text{Check: } x = -4$$

$$\log_{10}[(-4)^2 - (-4) - 19] = 0$$

$$\log_{10}(16 + 4 - 19) = 0$$

$$\log_{10}(25 - 5 - 19) = 0$$

$$\log_{10}(1) = 0$$

$$1 = 10^0 \quad \text{Yes}$$

Solution(s):  $x = 5$  or  $x = -4$

### Example 5

Solve for x:

$$\log_3(x+6) + \log_3 x = 3 \quad \text{use: } \log_b(x) + \log_b(y) = \log_b(x \cdot y)$$

$$\log_3[(x+6) \cdot x] = 3$$

$$\log_3[x^2 + 6x] = 3 \quad \text{use: } \log_b x = a \Rightarrow x = b^a$$

$$x^2 + 6x = 3^3$$

$$x^2 + 6x = 27$$

$$x^2 + 6x - 27 = 0$$

$$(x+9)(x-3) = 0$$

$$x = -9 \text{ or } x = 3$$

Check:  $x = -9$

$$\log_3(-9+6) + \log_3(-9) = 3$$

STOP:  $\log(\text{a negative number})$

$$x \neq -9$$

Check:  $x = 3$

$$\log_3(3+6) + \log_3(3) = 3$$

$$\log_3(9) + \log_3(3) = 3$$

$$\log_3(27) = 3$$

$$27 = 3^3$$

$$27 = 27 \quad \text{Yes}$$

Solution(s):  $x = 3$

### Example 6

Solve for x:

$$\log_2(5x - 6) - \log_2(x) = 3 \quad \text{use: } \log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

$$\log_2\left(\frac{5x - 6}{x}\right) = 3 \quad \text{use: } \log_b x = a \Rightarrow x = b^a$$

$$\left(\frac{5x - 6}{x}\right) = 2^3$$

$$\left(\frac{5x - 6}{x}\right) = 8 \quad \text{(multiply both sides by x)}$$

$$5x - 6 = 8x$$

$$-6 = 3x$$

$$x = -2$$

Check:  $x = -2$

$$\log_2[5(-2) - 6] - \log_2(-3) = 3$$

$$\log_2[-16] - \log_2(-2) = 3$$

STOP: log( a negative number)

$$x \neq -2$$

No Solution

### Example 7

Solve for x:

$$\log_4(2x) - \log_4(x - 3) = 1 \quad \text{use: } \log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

$$\log_4\left(\frac{2x}{x-3}\right) = 1 \quad \text{use: } \log_b x = a \Rightarrow x = b^a$$

$$\left(\frac{2x}{x-3}\right) = 4^1$$

$$\left(\frac{2x}{x-3}\right) = 4 \quad \text{(multiply both sides by } x - 3)$$

$$2x = 4x - 12$$

$$-2x = -12$$

$$x = 6$$

Check:  $x = 6$

$$\log_4(2x) - \log_4(x - 3) = 1$$

$$\log_4[2(6)] - \log_4[6 - 3] = 1$$

$$\log_4(12) - \log_4(3) = 1$$

$$\log_4\left(\frac{12}{3}\right) = 1$$

$$\log_4(4) = 1$$

$$4 = 4^1 \quad \text{Yes}$$

Solution:  $x = 6$