Section 5 – 5B: Properties of Logarithms

Express as the sum, difference and/or product

Use the following Property of Logarithmic Functions

$$\log_b(x) + \log_b(y) = \log_b(x \bullet y)$$

to express the sum of logarithms as single log function

Example 1

$\log_b(x) + \log_b(x+1)$

can be expressed as
the single log function

$\log_b\left[(x) \bullet (x+1)\right]$

$\log_b\left(x^2 + x\right)$

Example 2

$\log_b(4x) + \log_b(x+3)$

can be expressed as
the single log function

$\log_b\left[4x \bullet (x+3)\right]$

$\log_b\left(4x^2 + 12x\right)$

Use the following Property of Logarithmic Functions

$$\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

to express the the difference of logarithms as a single log function

Example 3

$\log_b(x-3) - \log_b(x+4)$

can be expressed as
the single log function

$\log_b\left(\frac{x-3}{x+4}\right)$

Example 4

$\log_b(\sqrt{x-2}) - \log_b(5)$

can be expressed as
the single log function

$\log_b\left(\frac{\sqrt{x-2}}{5}\right)$

Use the following Property of Logarithmic Functions

$$a \bullet \log_b(x) = \log_b(x)^a$$

to express the product of logarithms as a single log function

Example 5

$$4\log_2(2x-3)$$

can be expressed as
the single log function

$$= \log_2(2x-3)^4$$

Example 6

$$\frac{1}{2} \bullet \log_6(x-2)$$

can be expressed as
the single log function

$$= \log\ (x-2)^{1/2}$$

$$\log_6 \sqrt{x-2}$$

Use the Properties of Logarithmic Functions to express the sum, difference and/or products of logarithms as a single log function

Example 10

$3\log_5(4) + 2\log_5(x)$ use: $a \bullet \log_b(x) = \log_b(x)^a$

$\log_5(4^3) + \log_5(x^2)$ use: $\log_b(x) + \log_b(y) = \log_b(x \bullet y)$

$$\log_3(64 \bullet x^2)$$

$$= \log_3(64x^2)$$

Example 11

$2\log x + \log(x-3)$ use: $a\log_b(x) = \log_b(x)^a$

$\log(x^2) + \log(x-3)$ use: $\log_b(x) + \log_b(y) = \log_b(x \bullet y)$

$$= \log_3(x^3 - 3x)$$

Example 12

$2\log_b(5) - 3\log_b(x-2)$ use: $a \bullet \log_b(x) = \log_b(x)^a$

$\log_b(5^2) - \log_b(x-2)^3$

$\log_b(25) - \log_b(x-2)^3$ use: $\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$

$\log_b\left(\frac{25}{(x-2)^3}\right)$

Example 14

$3\log_b(2) + 3\log_b(y+4) - \frac{1}{2}(x-9)$ use: $a \bullet \log_b(x) = \log_b(x)^a$

$\log_b(5^2) + \log_b(y+4)^3 - (x-9)^{1/2}$

$\log_b(25) + \log_b(y+4)^3 - \sqrt{x-9}$ use: $\log_b(x) + \log_b(y) = \log_b(x \bullet y)$

and $\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$

$\log_b\left(\frac{25(y+4)^3}{\sqrt{x-9}}\right)$

$$\log_b\left(\frac{25x^7}{\sqrt{y}}\right)$$ use: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

$$\log_b\left(25x^7\right) - \log_b\left(\sqrt{y}\right)$$ use: $\log_b(x \bullet y) = \log_b(x) + \log_b(y)$

$$\log_b(25) + \log_b\left(x^7\right) - \log_b\left(\sqrt{y}\right)$$ Note: $25 = 5^2$ and $\sqrt{y} = y^{1/2}$

$$\log_b\left(5^2\right) + \log_b\left(x^7\right) - \log_b\left(y^{1/2}\right)$$ use: $\log_b(x)^a = a \bullet \log_b(x)$

$$= 2\log_b(5) + 7\log_b\left(x^7\right) - \frac{1}{2}\log_b(y)$$