Section 5 – 5A: Properties of Logarithms

Express as the sum, difference and/or product

Use the following Property of Logarithmic Functions

$$\log_b(x \bullet y) = \log_b(x) + \log_b(y)$$

to express the single log function as the sum of logarithms

Example 1

$\log_b(x \bullet y)$

can be expressed as the sum

$= \log_b(x) + \log_b(y)$

Example 2

$\log_2(5 \bullet x)$

can be expressed as the sum

$= \log_2(5) + \log_2(x)$

Example 3

$\log_b(2 \bullet x \bullet w)$

can be expressed as the sum

$= \log_2(2) + \log_2(x) + \log_2(y)$

Use the following Property of Logarithmic Functions

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

to express the single log function as the difference of logarithms

Example 4

$\log_b\left(\frac{2}{x}\right)$

can be expressed as the difference

$= \log_b(2) - \log_b(x)$

Example 5

$\log_5\left(\frac{y}{3}\right)$

can be expressed as the difference

$= \log_5(y) - \log_b(3)$

Example 6

$\ln\left(\frac{x+1}{y}\right)$

can be expressed as the difference

$= \ln(x+1) - \ln(y)$

Use the following Property of Logarithmic Functions

$$\log_b (x)^a = a \bullet \log_b (x)$$

to express the single log function as the product of logarithms

Example 7

$\log_b x^5$

can be expressed as the product

$= 5 \bullet \log_b (x)$

Example 8

$\ln(x - 1)^3$

can be expressed as the product

$= 3 \bullet \ln(x - 1)$

Note: $\ln(x-1)$ is $\log_e (x-1)$

Example 9

$\log(2x + 3)^5$

can be expressed as the product

$= 5 \bullet \log(2x + 3)$

Note: The base is 10

Use the Properties of Logarithmic Functions to express each single log function as the sum, difference and/or product

Example 10

$\log_3 \left(x^2 y^3\right)$ use: $\log_b (x \bullet y) = \log_b (x) + \log_b (y)$

$\log_3 \left(x^2\right) + \log_3\left(y^3\right)$ use: $\log_b (x)^a = a \bullet \log_b (x)$

$= 2\log_3 (x) + 3\log_3 (y)$

Example 11

$\log_3 \left(x^5 \sqrt{y}\right)$ use: $\log_b (x \bullet y) = \log_b (x) + \log_b (y)$

$\log_3 \left(x^2\right) + \log_3\left(y^{1/2}\right)$ use: $\log_b (x)^a = a \bullet \log_b (x)$

$= 5 \log_3 (x) + \frac{1}{2}\log_3 (y)$

Example 12

$\log_b\left(81x^5\right)$ use: $\log_b(x \bullet y) = \log_b(x) + \log_b(y)$

$\log_b\left(81\right) + \log_b\left(x^5\right)$ Note: $61 = 3^4$

$\log_b\left(3^4\right) + \log_b\left(x^5\right)$ use: $\log_b\left(x\right)^a = a \bullet \log_b(x)$

$= 4\log_b\left(3\right) + 5\log_b\left(x\right)$

Example 13

$\log_b\left(\frac{x^2}{y^3}\right)$ use: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

$\log_b\left(x^2\right) - \log_b\left(y^3\right)$ use: $\log_b\left(x\right)^a = a \bullet \log_b(x)$

$2\log_b\left(x\right) - 3\log_b\left(y\right)$

$= 4\log_b\left(3\right) - 3\log_b\left(x\right)$

Example 14

$\log_b\left(\frac{25x^7}{\sqrt{y}}\right)$ use: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

$\log_b\left(25x^7\right) - \log_b\left(\sqrt{y}\right)$ use: $\log_b(x \bullet y) = \log_b(x) + \log_b(y)$

$\log_b\left(25\right) + \log_b\left(x^7\right) - \log_b\left(\sqrt{y}\right)$ Note: $25 = 5^2$ and $\sqrt{y} = y^{1/2}$

$\log_b\left(5^2\right) + \log_b\left(x^7\right) - \log_b\left(y^{1/2}\right)$ use: $\log_b\left(x\right)^a = a \bullet \log_b(x)$

$= 2\log_b\left(5\right) + 7\log_b\left(x^7\right) - \frac{1}{2}\log_b\left(y\right)$