Section 5 – 2B: Graphs of Decreasing Exponential Functions

We want to determine what the graph of an exponential function

$$y = a^x$$

looks like for all values of a such that 0 > a > 1

We will select a value of a such that 0 > a > 1 and examine several ordered pairs

We will use a = $\frac{1}{2}$

The patterns we find for a = 1/2 will be true for any value of a such that 0 > a > 1

The graph of $y = \left(\frac{1}{2}\right)^x$

x and y values for the right side of the graph

if $x = 0$	if $x = 1$	if $x = 2$	if $x = 3$	if $x = 4$	if $x = 5$
for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$
$y = \left(\frac{1}{2}\right)^0$	$y = \left(\frac{1}{2}\right)^1$	$y = \left(\frac{1}{2}\right)^2$	$y = \left(\frac{1}{2}\right)^3$	$y = \left(\frac{1}{2}\right)^4$	$y = \left(\frac{1}{2}\right)^5$
$y = 1$	$y = \frac{1}{2}$	$y = \frac{1}{4}$	$y = \frac{1}{8}$	$y = \frac{1}{16}$	$y = \frac{1}{32}$

As the values for x become larger and larger positive values the values for y get **closer and closer to 0.** These y values will never have a value of 0 but the y values will continue to get closer and closer to 0 as the graph continues to the left. The **right side of the graph will get closer and closer to the x axis.** We call the line that the graph approaches but does not reach an **asymptote**. We use a **dotted line** to show the asymptotic line.

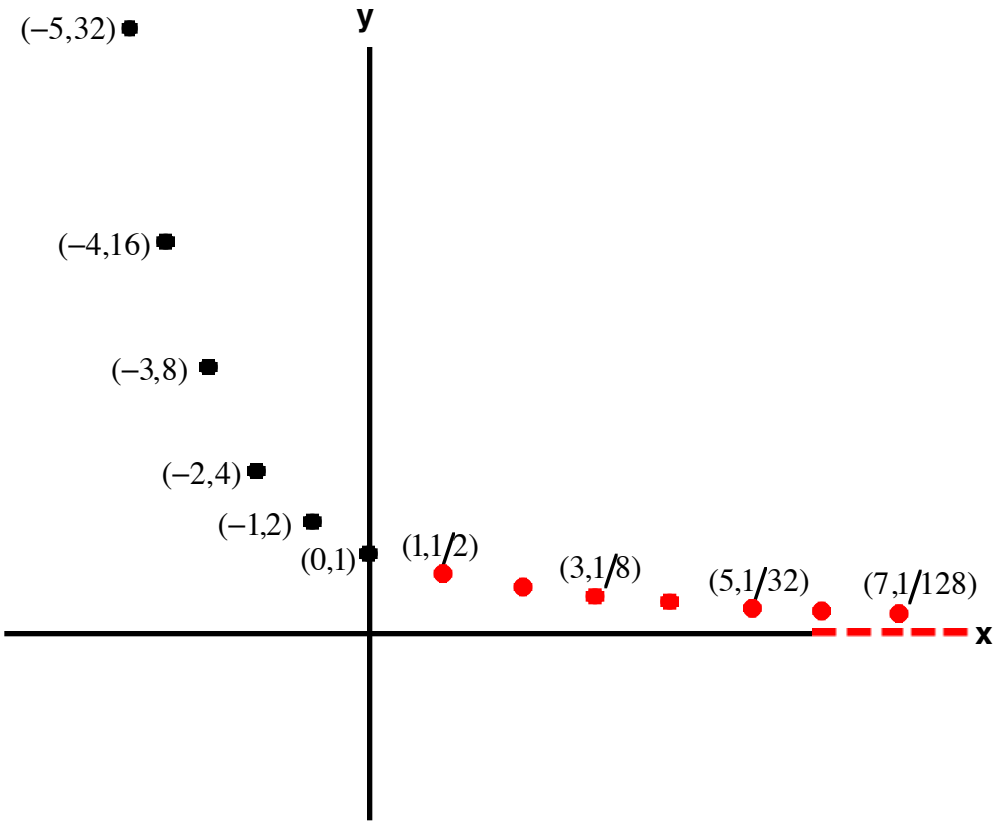
x and y values for the left side of the graph

if $x = -6$	if $x = -5$	if $x = -4$	if $x = -3$	if $x = -2$	if $x = -1$
for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$	for $y = \left(\frac{1}{2}\right)^x$
$y = \left(\frac{1}{2}\right)^{-6}$	$y = \left(\frac{1}{2}\right)^{-5}$	$y = \left(\frac{1}{2}\right)^{-4}$	$y = \left(\frac{1}{2}\right)^{-3}$	$y = \left(\frac{1}{2}\right)^{-2}$	$y = \left(\frac{1}{2}\right)^{-1}$
$y = 64$	$y = 32$	$y = 16$	$y = 8$	$y = 4$	$y = 2$

As the values for **x become larger and larger negative numbers** then the values for **y become larger and larger positive values.** This means that the **left end of the graph is curve pointing up and to the left**

The table below shows several values for pairs of (x, y) for $\mathbf{y = (1/2)^x}$

x	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
y	256	128	64	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$



The graph of $y = \left(\frac{1}{2}\right)^x$

1. **Domain – Range:** x can be any real number so the **domain of the function is all real numbers. The range for y are positive numbers greater than 0.** This means that the entire graph of the function will be above the x axis.

2. The **y intercept is at (0 , 1)**

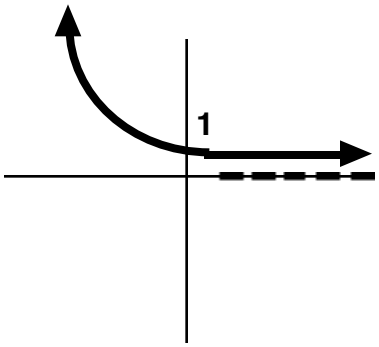
3. **Left end of the graph**: As the values for **x become larger and larger negative numbers** then the values for **y become larger and larger positive values.** This means that the **left end of the graph is curve pointing up and to the left.**

4. **Right end of the graph:** As the values for x become larger and larger positive values the values for y get **closer and closer to 0.** These y values will never have a value of 0 but the y values will continue to get closer and closer to 0 as the graph continues to the left. The **right side of the graph will get closer and closer to the x axis.** We call the line that the graph approaches but does not reach an **asymptote**. We use a **dotted line** to show the asymptotic line.

The Graph of a Decreasing Exponential Function

The graph of $y = a^x$

for all values of a such that $0 > a > 1$ is shown below.



$y = a^x$ for all values of a such that $0 > a > 1$ is called an **decreasing exponential function**

1. **Domain – Range:** x can be any real number so the **domain of the function is all real numbers. The range for y are positive numbers greater than 0 or $y > 0$.** This means that the entire graph of the function will be above the x axis.

2. The **y intercept is at (0 , 1)**

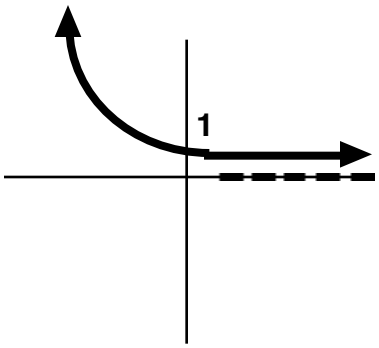
3. The **x axis is a horizontal asymptote**. We use a **dotted line** to show the asymptotic line. Every increasing logarithmic function has a horizontal asymptote.

4. **Left end of the graph**: As the values for **x become larger and larger negative numbers** then the values for **y become larger and larger positive values.** This means that the **left end of the graph is curve pointing up and to the left.**

5. **Right end of the graph:** As the values for x become larger and larger positive values the values for y get **closer and closer to 0.** These y values will never have a value of 0 but the y values will continue to get closer and closer to 0 as the graph continues to the left. The **right side of the graph will get closer and closer to the x axis.** We call the line that the graph approaches but does not reach an **asymptote**. We use a **dotted line** to show the asymptotic line.

There are 6 different transformation that change the position of the graph, asymptote and y intercept. The graph of $y = (1/2)^x$ and be moved RiGHT, moved LEFT, moved UP, moved Down, flipped about the x AXIS or flipped about the y AXIS. These transformations are effected by the addition, subtraction or multiplication of various part of the equation.

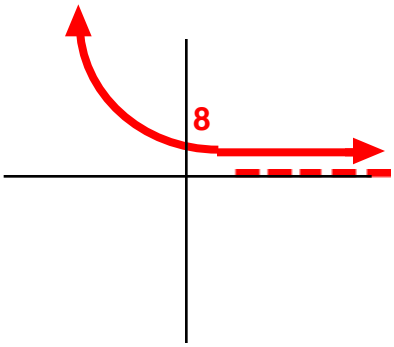
Translating the graph of $y = \left(\frac{1}{2}\right)^x$



Translation 1:

$y = (1/2)^{x-3}$

subtracting 3 from x
moves the graph **RIGHT 3**



The y intercept is found by letting x = 0 and finding y

$y = (1/2)^{x-3}$

$y = (1/2)^{(0-3)}$

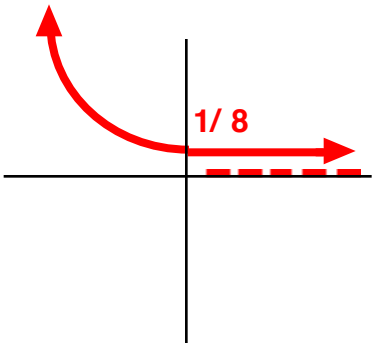
$y = (1/2)^{-3}$

$y = 8$

Translation 2:

$y = (1/2)^{x+3}$

adding 3 to x
moves the graph **LEFT 3**



The y intercept is found by letting x = 0 and finding y

$y = (1/2)^{x+3}$

$y = (1/2)^{0+3}$

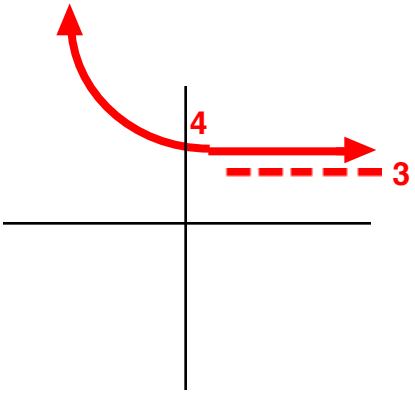
$y = (1/2)^{+3}$

$y = 1/8$

Translation 3:

$y = (1/2)^{x+3}$

Adding 3 at the end
moves the graph **UP 3**



The y intercept is found by letting x = 0 and finding y

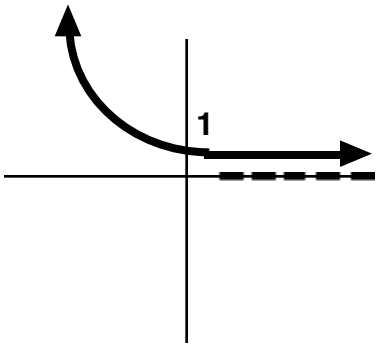
$y = (1/2)^{x+3}$

$y = (1/2)^{0+3}$

$y = 1+3$

$y = 4$

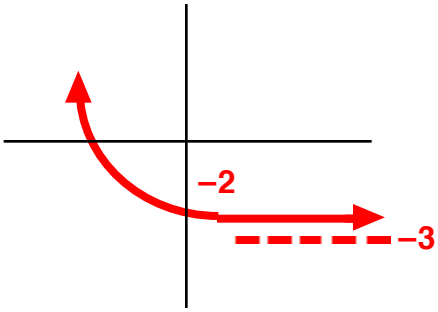
Translating the graph of $y = \left(\frac{1}{2}\right)^x$



Translation 4:

$y = (1/2)^x - 3$

subtracting 3 at the end moves the graph **DOWN 3**



The y intercept is found by letting x = 0 and finding y

$y = (1/2)^x - 3$

$y = (1/2)^0 - 3$

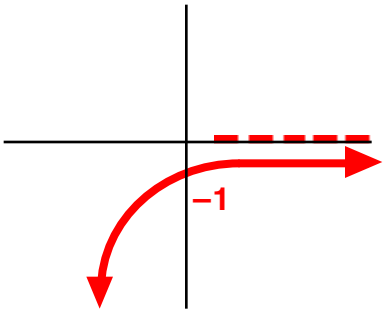
$y = 1 - 3$

$y = -2$

Translation 5: x

$y = -(1/2)^x$

negating y flips the graph about the x axis



The y intercept is found by letting x = 0 and finding y

$y = -(1/2)^x$

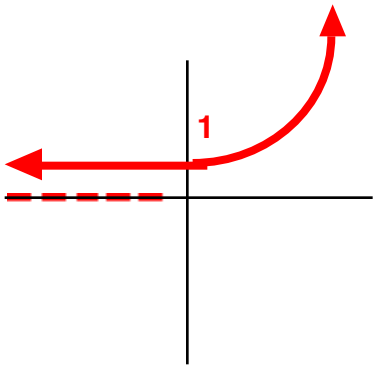
$y = -(1/2)^0 = -1$

$y = -1$

Translation 6:

$y = (1/2)^{-x}$

negating x flips the graph about the y axis



The y intercept is found by letting x = 0 and finding y

$y = (1/2)^{-x}$

$y = (1/2)^{-0}$

$y = 2^0$

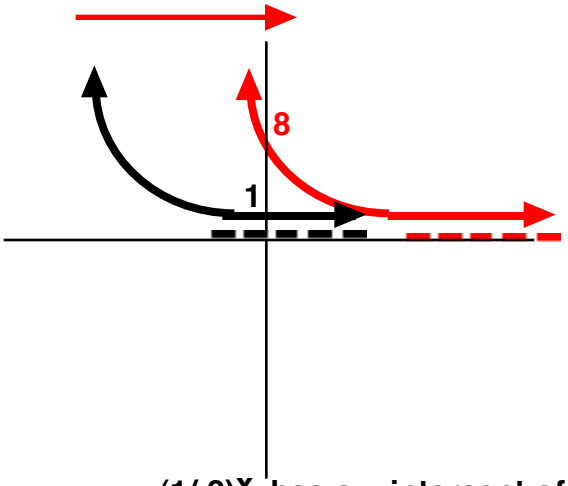
$y = 1$

The graph of $y = (1/2)^{x-3}$ compared to the graph of $y = (1/2)^x$

$y = (1/2)^{x-3}$ is equivalent to $y = (1/2)^{(x-3)}$

Subtracting 3 from the x inside a bracket moves the graph 3 units to the RIGHT

compared to $y = (1/2)^x$ $y = (1/2)^{x-3}$ **moves 3 to the right**



$y = (1/2)^x$ has a y intercept of 1

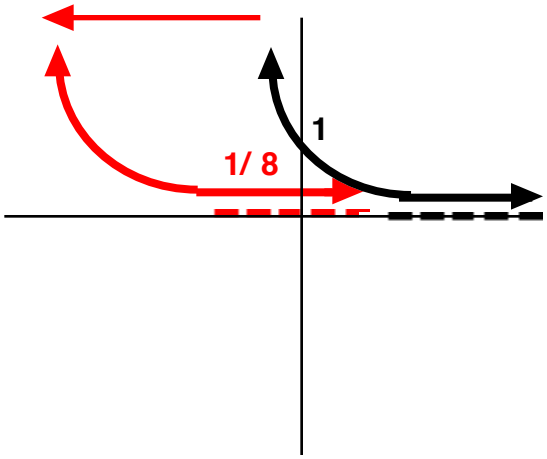
$y = (1/2)^{x-3}$ has a y intercept of 8

The graph of $y = (1/2)^{x+3}$ compared to the graph of $y = (1/2)^x$

$y = 2^{x+3}$ is equivalent to $y = 2^{(x+3)}$

Adding 3 to the x inside a bracket moves the graph 3 units to the LEFT

$y = (1/2)^{x+3}$ **moves 3 to the left** **compared to $y = (1/2)^x$**



$y = (1/2)^x$ has a y intercept of 1

$y = (1/2)^{x+3}$ has a y intercept of 1/ 8

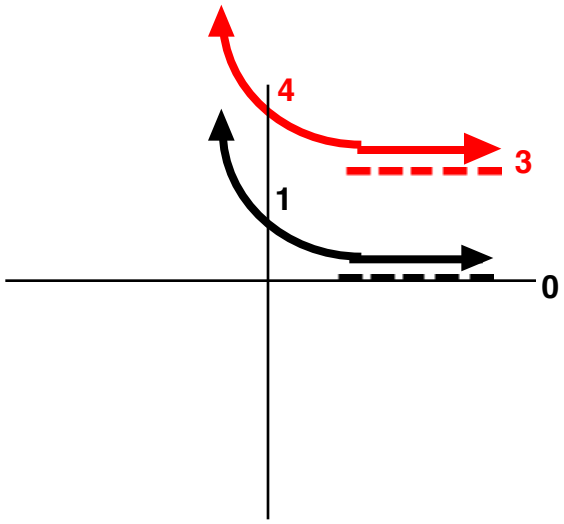
The graph of $y = (1/2)^x + 3$ compared to the graph of $y = (1/2)^x$

$y = (1/2)^x + 3$

Adding 3 to the $(1/2)^x$ at the end of the equation moves the graph UP 3 units

$y = (1/2)^x + 3$ moves up 3

compared to $y = (1/2)^x$



$y = (1/2)^x$ has a y intercept of 1

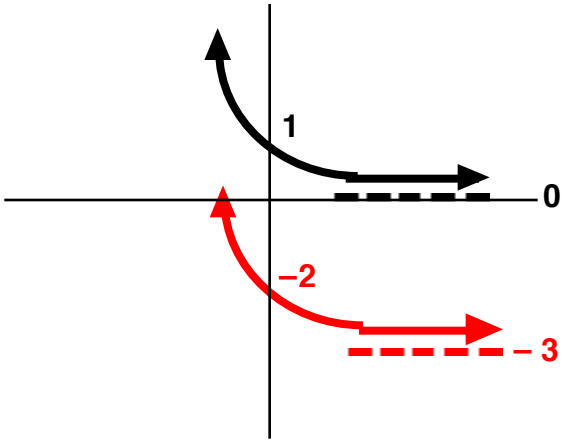
$y = (1/2)^x + 3$ has a y intercept of 4

The graph of $y = (1/2)^x - 3$ compared to the graph of $y = (1/2)^x$

Subtracting 3 to the $(1/2)^x$ at the end of the equation moves the graph DOWN 3 units

compared to $y = (1/2)^x$

$y = (1/2)^x - 3$ moves down 3



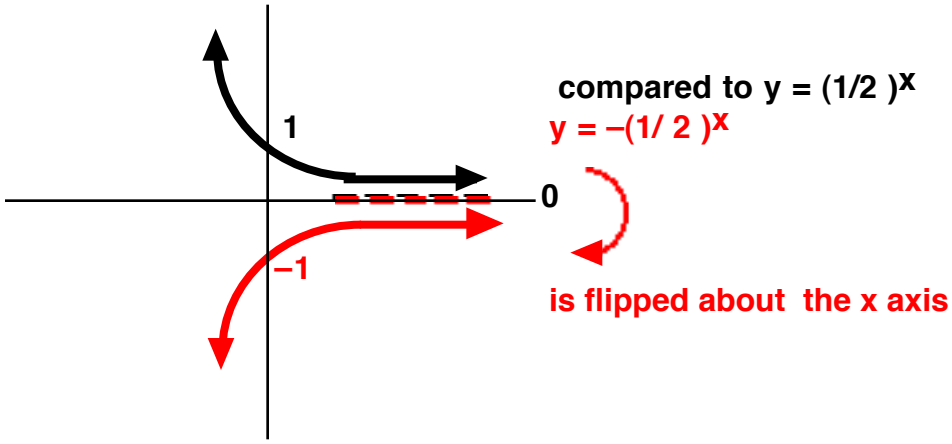
$y = (1/2)^x$ has a y intercept of 1

$y = (1/2)^x - 3$ has a y intercept of –2

The graph of $y = -(1/2)^x$ compared to the graph of $y = (1/2)^x$

Multiplying the $(1/2)^x$ by –1 is equivalent to negating the y values

(**negating the y values flips the graph about the x axis**)



$y = (1/2)^x$ has a y intercept of 1

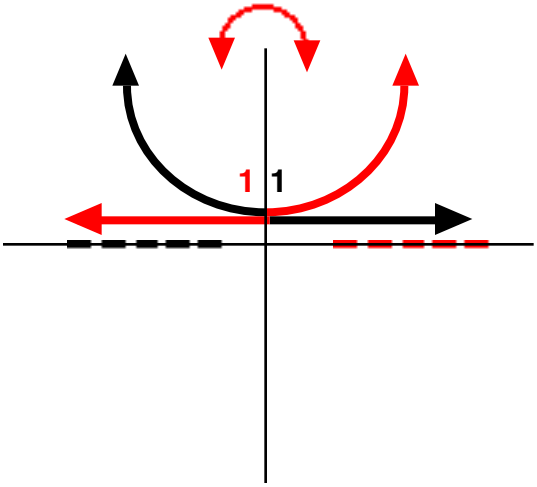
$y = -(1/2)^x$ has a y intercept of –1

The graph of $y = (1/2)^x$ compared to the graph of $y = (1/2)^{-x}$

Multiplying the x in the exponent by a negative one has the effect of flipping the graph about the y

(**negating the x values flips the graph about the y axis**)

compared to $y = (1/2)^x$ $y = (1/2)^{-x}$ **is flipped about the y axis**



$y = (1/2)^x$ has a y intercept of 1

$y = (1/2)^{-x}$ has a y intercept of 1