Section 5 – 2A: Graphs of Increasing Exponential Functions

We want to determine what the graph of an exponential function

$y = a^x$

looks like for all values of a > 1

We will select a value of a > 1 and examine several ordered pairs for

$y = a^x$

We will use a = 2

The patterns we find for a = 2 will be true for any value of a > 1

The graph of $y = 2^x$

x and y values for the right side of the graph

if $x = 0$	if $x = 1$	if $x = 2$	if $x = 3$	if $x = 4$	if $x = 5$
for $y = 2^x$	for $y = 2^x$	for $y = 2^x$	for $y = 2^x$	for $y = 2^x$	for $y = 2^x$
$y = 2^0$	$y = 2^1$	$y = 2^2$	$y = 2^3$	$y = 2^4$	$y = 2^5$
$y = 1$	$y = 2$	$y = 4$	$y = 8$	$y = 16$	$y = 32$

As the values for **x become larger and larger positive numbers** then the values for **y become larger and larger positive values.** This means that the r**ight end of the graph is a slowly increasing curve pointing up and to the right**

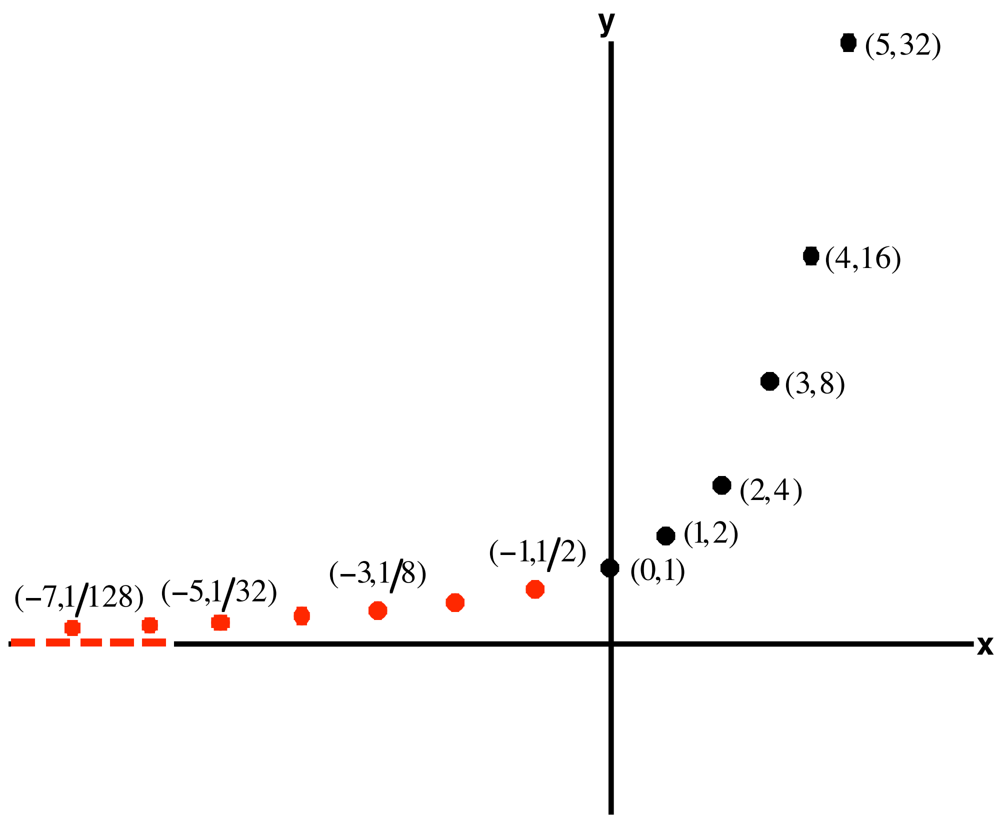
x and y values for the left side of the graph

if $x = -6$	if $x = -5$	if $x = -4$	if $x = -3$	if $x = -2$	if $x = -1$
for $y = 2^x$	for $y = 2^x$	for $y = 2^x$	for $y = 2^x$	for $y = 2^x$	for $y = 2^x$
$y = 2^{-6}$	$y = 2^{-5}$	$y = 2^{-4}$	$y = 2^{-3}$	$y = 2^{-2}$	$y = 2^{-1}$
$y = \frac{1}{64}$	$y = \frac{1}{32}$	$y = \frac{1}{16}$	$y = \frac{1}{8}$	$y = \frac{1}{4}$	$y = \frac{1}{2}$

As the values for x become larger and larger negative values the values for y get **closer and closer to 0.** These y values will never have a value of 0 but the y values will continue to get closer and closer to 0 as the graph continues to the left. The **left side of the graph will get closer and closer to the x axis.** We call the line that the graph approaches but does not reach an **asymptote**. We use a **dotted line** to show the asymptotic line.

The table below shows several values for pairs of (x, y) for **y = 2^{x}**

x	−8	−7	−6	−5	−4	−5	−2	−1	0	1	2	3	4	5	6	7	8
y	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64	128	256



The graph of **y = 2^{x}**

1. **Domain – Range:** x can be any real number so the **domain of the function is all real numbers. The range for y are positive numbers greater than 0.** This means that the entire graph of the function will be above the x axis.

2. The **x intercept is at (1 , 0)**

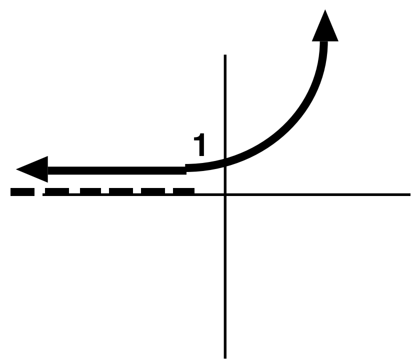
3. **Left end of the graph**: As the values for x become larger and larger negative values the values for y get **closer and closer to 0.** These y values will never have a value of 0 but the y values will continue to get closer and closer to 0 as the graph continues to the left. We draw the graph of the left end of the graph getting closer and closer to the x axis without touching the x axis. We call the line that the graph approaches but does not reach an **asymptote**. We use a **dotted line** to show the asymptotic line.

4. **Right end of the graph:** As the values for **x become larger and larger positive numbers** then the values for **y become larger and larger positive values.** As the values for x increase the y values get extremely large. We show the right end of the graph pointing upwards and to the right and we use an arrow to show that it continues to increase.

The Graph of a Increasing Exponential Function

The graph of $y = a^x$

for all values of a > 1 is shown below.



$y = a^x$ for all values of a > 1 is called an **increasing exponential function**

1. **Domain – Range:** x can be any real number so the **domain of the function is all real numbers. The range for y are positive numbers greater than 0 or y > 0 .** This means that the entire graph of the function will be above the x axis.

2. The **x intercept is at (1 , 0)**

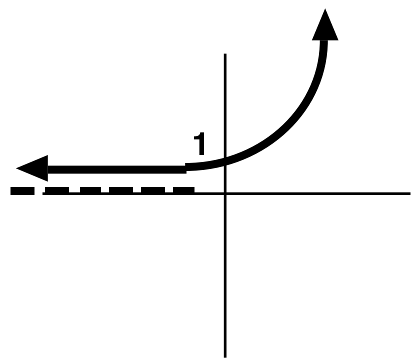
3. The **x axis is a horizontal asymptote**. We use a **dotted line** to show the asymptotic line. Every increasing logarithmic function has a horizontal asymptote.

3. **Left end of the graph**: As the values for x become larger and larger negative values the values for y get **closer and closer to 0.** These y values will never have a value of 0 but the y values will continue to get closer and closer to 0 as the graph continues to the left. The **left side of the graph will get closer and closer to the x axis.** We show the graph of the left end of the graph getting closer and closer to the x axis without touching the x axis. We call the line that the graph approaches but does not reach an **asymptote**. We use a **dotted line** to show the asymptotic line.

4. **Right end of the graph:** As the values for **x become larger and larger positive numbers** then the values for **y become larger and larger positive values.** As the values for x increase the y values get extremely large. We show the right end of the graph pointing upwards and to the right and we use an arrow to show that it continues to increase.

There are 6 different transformation that change the position of the graph, asymptote and y intercept. The graph of $\mathbf{y = 2^x}$ and be moved RiGHT, moved LEFT, moved UP, moved Down, flipped about the X AXIS or flipped about the Y AXIS. These transformations are effected by the addition, subtraction or multiplication of various part of the equation.

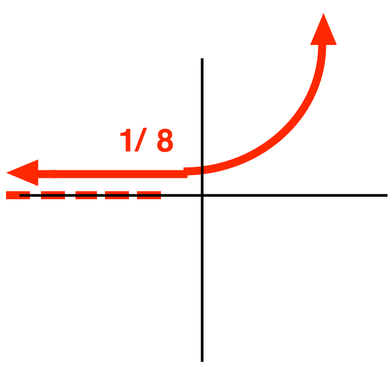
Translating the graph of $\mathbf{y = 2^x}$



Translation 1:

$y = 2^{x-3}$

subtracting 3 from x
moves the graph **RIGHT 3**



The y intercept is found by letting x = 0 and finding y

$y = 2^{x-3}$

$y = 2^{0-3}$

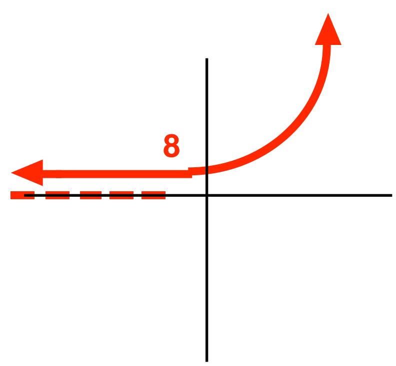
$y = 2^{-3}$

$y = 1/8$

Translation 2:

$y = 2^{x+3}$

adding 3 to x
moves the graph **LEFT 3**



The y intercept is found by letting x = 0 and finding y

$y = 2^{x+3}$

$y = 2^{0+3}$

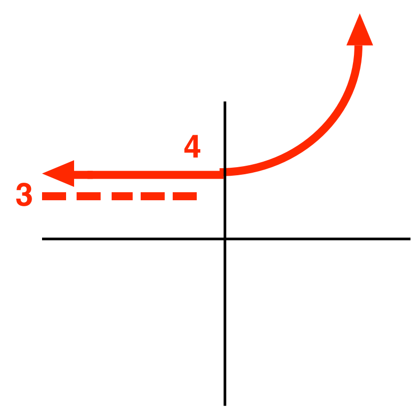
$y = 2^{3}$

$y = 8$

Translation 3:

$y = 2^x + 3$

Adding 3 at the end
moves the graph **UP 3**



The y intercept is found by letting x = 0 and finding y

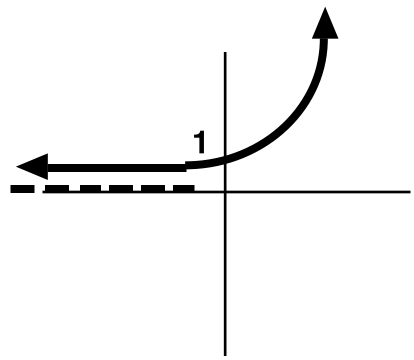
$y = 2^x + 3$

$y = 2^0 + 3$

$y = 1 + 3$

$y = 4$

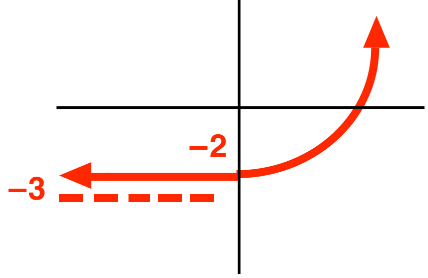
Translating the graph of **$y = 2^x$**



Translation 4:

$y = 2^x - 3$

subtracting 3 at the end moves the graph **DOWN 3**



The y intercept is found by letting x = 0 and finding y

$y = 2^x - 3$

$y = 2^0 - 3$

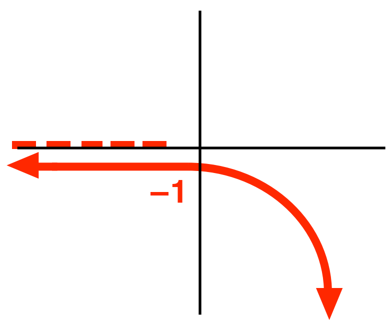
$y = 1 - 3$

$y = -2$

Translation 5:

$y = -2^x$

negating y **flips the graph about the x axis**



The y intercept is found by letting x = 0 and finding y

$y = -2^x$

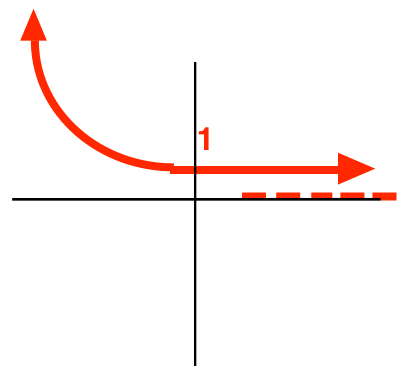
$y = -2^0$

$y = -1$

Translation 6:

$y = 2^{-x}$

negating x **flips the graph about the y axis**



The y intercept is found by letting x = 0 and finding y

$y = 2^{-x}$

$y = 2^{-0}$

$y = 2^0$

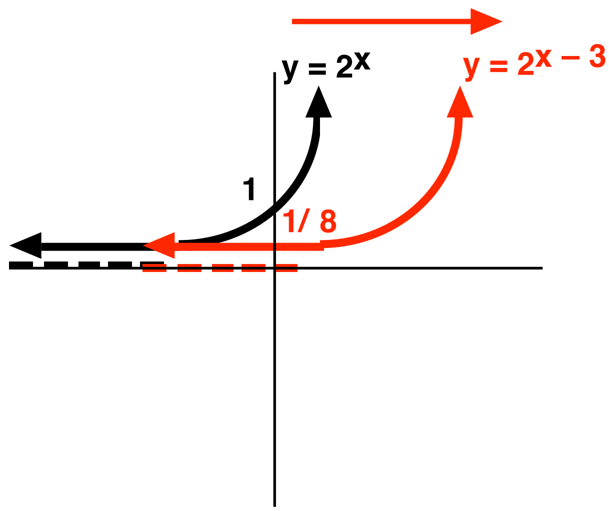
$y = 1$

The graph $y = 2^{x-3}$ compared to the graph of $y = 2^x$

$y = 2^{x-3}$ is equivalent to $y = 2^{(x-3)}$

Subtracting 3 from the x inside a bracket moves the graph 3 units to the RIGHT

compared to $y = 2^x$ the graph of $y = 2^{x-3}$ moves 3 to the right



$y = 2^x$ has a y intercept of 1

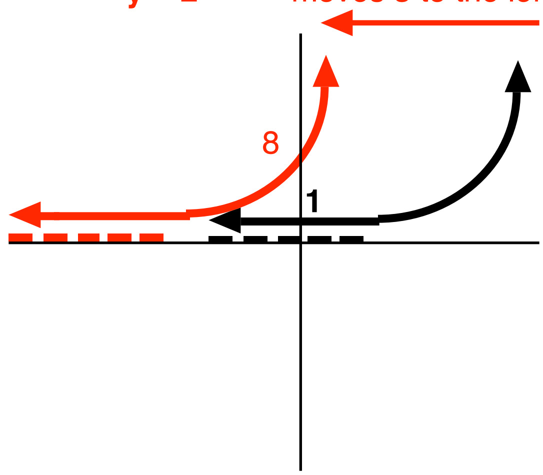
$y = 2^{x-3}$ has a y intercept of 1/ 8

The graph of $y = 2^{x+3}$ compared to the graph of $y = 2^x$

$y = 2^{x+3}$ is equivalent to $y = 2^{(x+3)}$

Adding 3 to the x inside a bracket moves the graph 3 units to the LEFT

$y = 2^{x+3}$ moves 3 to the left **compared to $y = 2^x$**



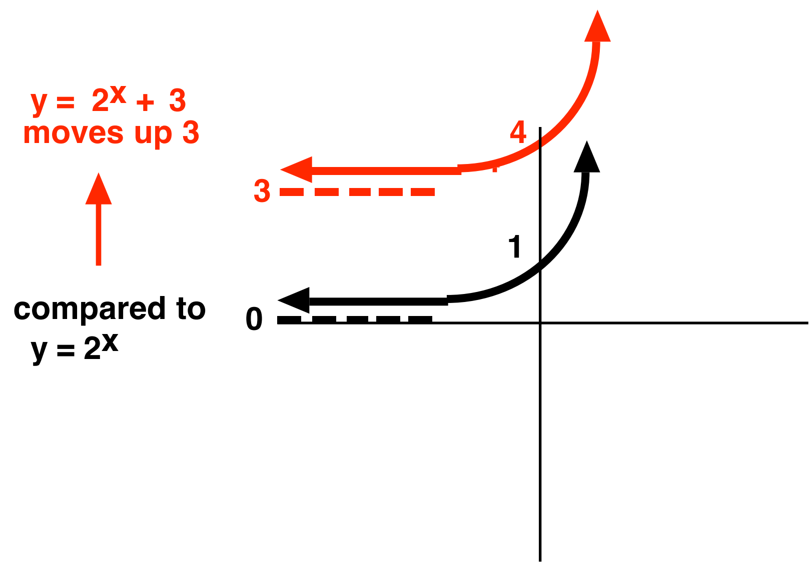
$y = 2^x$ has a y intercept of 1

$y = 2^{x+3}$ has a y intercept of 8

The graph of $y = 2^x + 3$ compared to the graph of $y = 2^x$

$y = 2^x + 3$ is equivalent to $y = (2^x) + 3$

Adding 3 to the 2^x at the end of the equation moves the graph UP 3 units



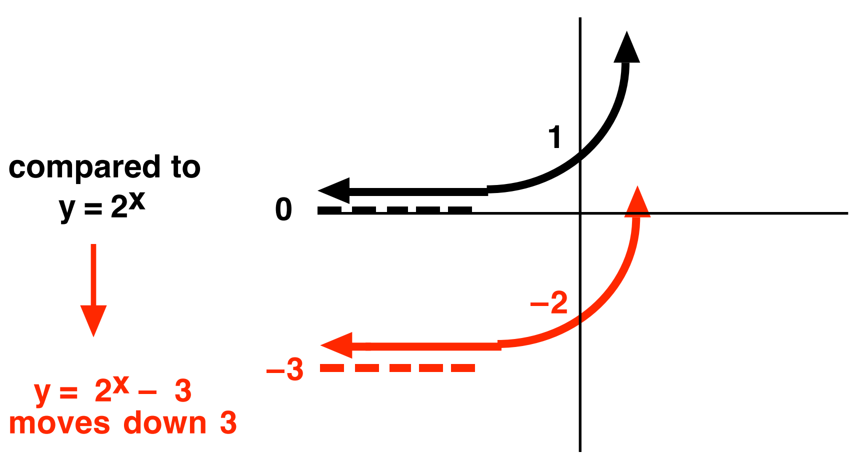
$y = 2^x$ has a y intercept of 1

$y = 2^x + 3$ has a y intercept of 4

The graph of $y = 2^x + 3$ compared to the graph of $y = 2^x$

$y = 2^x - 3$ is equivalent to $y = (2^x) - 3$

Subtracting 3 to the 2^x at the end of the equation moves the graph DOWN 3 units



$y = 2^x$ has a y intercept of 1

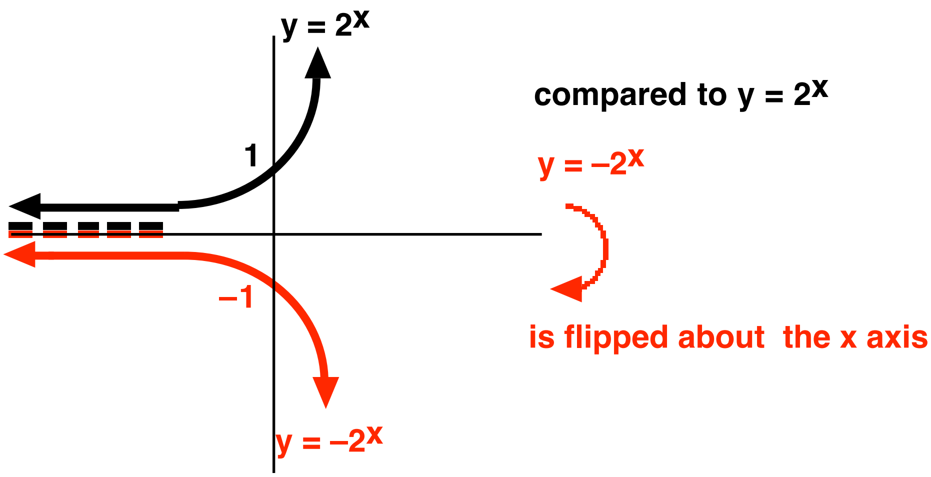
$y = 2^x - 3$ has a y intercept of –2

The graph of $y = -2^x$ compared to the graph of $y = 2^x$

Multiplying 2^x by -1 is equivalent to $(-1)\,2^x$ and is written as $y = -2^x$

Multiplying the 2^x by -1 is equivalent to negating the y values

(**negating the y values flips the graph about the x axis**)



$y = 2^x$ has a y intercept of 1

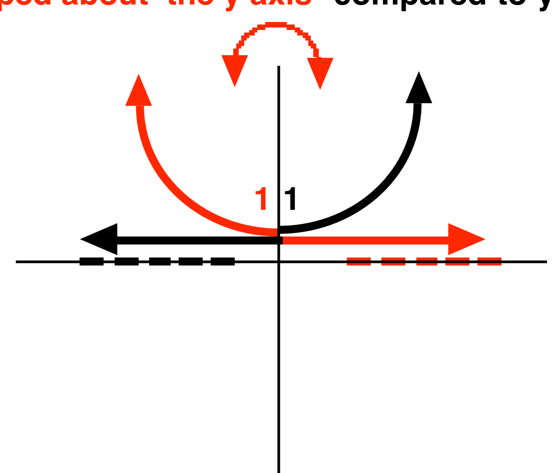
$y = -2^x$ has a y intercept of -1

The graph of $y = 2^{-x}$ compared to the graph of $y = 2^x$

Multiplying the x in the exponent by a negative one has the effect of flipping the graph about the y

(**negating the x values flips the graph about the y axis**)

$y = 2^{-x}$ is flipped about the y axis compared to $y = 2^x$



$y = 2^x$ has a y intercept of 1

$y = 2^{-x}$ has a y intercept of 1