Section 5 – 1: Solving Exponential Equations

Polynomial Equations are equations that have several different variable terms where each term is a different power of the variable. The variable is in the base and the exponent is a whole number.

$3x - 4 = 0$ is a first degree polynominal equation

$x^2 - 3x - 4 = 0$ is a second degree polynominal equation

$x^3 - 3x^2 - 4x - 6 = 0$ is a third degree polynominal equation

Exponential Equations

Exponential Equations are equations that have a rational number in the base and the **variable is in the exponent** instead of the base.

$3^x = 3^4$ is an Exponential Equation with a base of 3

$8^{x+2} = 8^4$ is an Exponential Equation with a base of 8

The solution to an exponential equation is a real number value for x that makes the Exponential Equation true. The Law of Exponents says that if the bases on both sides of the equation are the same then the exponents above the common base must be equal to each other. The use of the Law of Exponents allows us to eliminate the base and solve for x using the techniques of solving first degree equations.

The Law of Exponents

If $b^x = b^y$

then

$x = y$

for and real number where **$b \neq -1, 1,$ or 0**

Example 1

Solve the exponential equation for x

$$2^{x+3} = 2^5$$

The bases are both 2 so the exponential expressions must be equal to each other:

$$x + 3 = 5$$

$$x = 2$$

Check:

$$2^{2+3} = 2^5$$

$$2^5 = 2^5$$

Example 2

Solve the exponential equation for x

$$7^{3x+1} = 7^{13}$$

The bases are both 7 so the exponential expressions must be equal to each other:

$$3x + 1 = 13$$

$$x = 4$$

Check:

$$7^{3(4)+1} = 7^{13}$$

$$7^{13} = 7^{13}$$

If the bases are not equal then you must use the laws of exponents to get a common base on each side of the equation. It is often possible to state **one base as a power of the other base.**

Example 3

Solve the exponential equation for x

$$2^{x-1} = 16$$

The bases are not equal but $16 = 2^4$

so

$$2^{x-1} = 16$$

is rewritten as

$$2^{x-1} = 2^4$$

The bases are both 2 so

$$x - 1 = 4$$

$$x = 5$$

Check:

$$2^{5-1} = 2^4$$

$$2^4 = 2^4$$

Example 4

Solve the exponential equation for x

$$3^{2x-5} = 27$$

The bases are not equal but $27 = 3^3$

so

$$3^{2x-5} = 27$$

is rewritten as

$$3^{2x-5} = 3^3$$

The bases are both 3 so

$$2x - 5 = 3$$

$$x = 4$$

Check:

$$3^{2(4)-5} = 3^3$$

$$3^3 = 3^3$$

Example 5

Solve the exponential equation for x

$$5 = 5^{x+3}$$

The bases are not equal but $5 = 5^1$

so

$$5 = 5^{x+3}$$

is rewritten as

$$5^1 = 5^{x+3}$$

The bases are both 5 so

$$1 = x + 3$$

$$-2 = x$$

Check:

$$5^1 = 5^{-2+3}$$

$$5^1 = 5^1$$

Example 6

Solve the exponential equation for x

$$7^{x-4} = 1$$

The bases are not equal but $1 = 7^0$

so

$$7^{x-4} = 1$$

is rewritten as

$$7^{x-4} = 7^0$$

The bases are both 7 so

$$x - 4 = 0$$

$$x = 4$$

Check:

$$7^{4-4} = 7^0$$

$$7^0 = 7^0$$

Example 7

Solve the exponential equation for x

$$\frac{9}{25}=\left(\frac{3}{5}\right)^{x+1}$$

The bases are not equal but $\frac{9}{25}=\left(\frac{3}{5}\right)^2$

so

$$\frac{9}{25}=\left(\frac{3}{5}\right)^{x+1}$$

is rewritten as

$$\left(\frac{3}{5}\right)^2=\left(\frac{3}{5}\right)^{x+1}$$

The bases are both 3 / 5 so

$$2 = x + 1$$
$$1 = x$$

Check:

$$\frac{9}{25}=\left(\frac{3}{5}\right)^{1+1}$$
$$\frac{9}{25}=\left(\frac{3}{5}\right)^2$$

Example 8

Solve the exponential equation for x

$$\frac{3}{2}=\left(\frac{27}{8}\right)^{x}$$

The bases are not equal but $\frac{27}{8}=\left(\frac{3}{2}\right)^3$

so

$$\frac{3}{2}=\left(\frac{27}{8}\right)^{x}$$

is rewritten as

$$\frac{3}{2}^1=\left(\frac{3}{2}\right)^{3x}$$

The bases are both 3 / 2 so

$$1 = 3x$$
$$\frac{1}{3}=x$$

Check:

$$\frac{3}{2}=\left(\frac{27}{8}\right)^{\frac{1}{3}}$$
$$\frac{3}{2}=\frac{3}{2}$$

Example 9

Solve the exponential equation for x

$$\frac{27}{64} = \left(\frac{4}{3}\right)^x$$

The bases are not equal but $\frac{27}{64} = \left(\frac{3}{4}\right)^3$

a base of $\frac{4}{3}$ is required so the $\left(\frac{3}{4}\right)^3$

must be inverted (flipped)

$$\frac{27}{64} = \left(\frac{3}{4}\right)^3 = \left(\frac{4}{3}\right)^{-3}$$

so

$$\frac{27}{64} = \left(\frac{4}{3}\right)^x$$

is rewritten as

$$\left(\frac{4}{3}\right)^{-3} = \left(\frac{4}{3}\right)^x$$

The bases are both 3 / 2 so

$$-3 = x$$

Check:

$$\frac{27}{64} = \left(\frac{4}{3}\right)^{-3}$$

$$\frac{27}{64} = \left(\frac{3}{4}\right)^3$$

$$\frac{27}{64} = \frac{27}{64}$$

Example 10

Solve the exponential equation for x

$$\frac{4}{9} = \left(\frac{3}{2}\right)^{x+3}$$

The bases are not equal but $\frac{4}{9} = \left(\frac{2}{3}\right)^2$

a base of $\frac{3}{2}$ is required so the $\left(\frac{2}{3}\right)^2$

must be inverted (flipped)

$$\frac{4}{9} = \left(\frac{2}{3}\right)^2 = \left(\frac{3}{2}\right)^{-2}$$

so

$$\frac{4}{9} = \left(\frac{3}{2}\right)^{x+3}$$

is rewritten as

$$\left(\frac{3}{2}\right)^{-2} = \left(\frac{3}{2}\right)^{x+3}$$

The bases are both 3/2 so

$$-2 = x + 3$$

$$-5 = x$$

Check:

$$\frac{4}{9} = \left(\frac{3}{2}\right)^{-5+3}$$

$$\frac{4}{9} = \left(\frac{3}{2}\right)^{-2}$$

$$\frac{4}{9} = \left(\frac{4}{9}\right)$$

If both bases are not equal then both bases must be expressed as a power of the same base but that base will not be either of the bases shown in the problem. This will require that you **rewrite each of the different bases** as powers of the same number.

Example 11

Solve the exponential equation for x

$$9^x = 27$$

The bases are not equal but

$$9^x = \left(3^2\right)^x = 3^{2x} \quad \text{and} \quad 27 = 3^3$$

so

$$9^x = 27$$

is rewritten

$$3^{2x} = 3^3$$

The bases are both 3 so

$$2x = 3$$

$$x = \frac{3}{2}$$

Check:

$$9^{\frac{3}{2}} = 27$$

Example 12

Solve the exponential equation for x

$$16^x = 32$$

The bases are not equal but

$$16^x = \left(2^4\right)^x = 2^{4x} \quad \text{and} \quad 32 = 2^5$$

so

$$16^x = 32$$

is rewritten as

$$2^{4x} = 2^5$$

The bases are both 2 so

$$4x = 5$$

$$x = \frac{5}{4}$$

Check:

$$16^{\frac{5}{4}} = 32$$