

Limits Involving Vertical Asymptotes

The limits on the left and right sides of a vertical asymptote

$$\text{If } f(c) = \frac{\text{nonzero number}}{0} \Rightarrow \text{Vertical asymptote at } x = c$$

Left sided Limit

The Limit as $x \rightarrow c$ from the left

$$\lim_{x \rightarrow c^-} f(x) = \text{either } +\infty \text{ or } -\infty$$

depending of the sign of

$f(c - \text{a very small decimal number})$

If $f(c - \text{a very small decimal number})$

is positive then $\lim_{x \rightarrow c^-} f(x) = +\infty$

If $f(c - \text{a very small decimal number})$

is negative then $\lim_{x \rightarrow c^-} f(x) = -\infty$

Right sided Limit

The Limit as $x \rightarrow c$ from the right

$$\lim_{x \rightarrow c^+} f(x) = \text{either } +\infty \text{ or } -\infty$$

depending of the sign of

$f(c + \text{a very small decimal number})$

If $f(c + \text{a very small decimal number})$

is positive then $\lim_{x \rightarrow c^+} f(x) = +\infty$

If $f(c + \text{a very small decimal number})$

is negative then $\lim_{x \rightarrow c^+} f(x) = -\infty$

The Limit as $x \rightarrow c$ from both the left and right sides

$$\lim_{x \rightarrow c} f(x) = \text{either } +\infty \text{ or } -\infty \text{ or DNE}$$

depending of the values of

$$\lim_{x \rightarrow c^-} f(x) \quad \text{and} \quad \lim_{x \rightarrow c^+} f(x)$$

$$\text{if } \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = +\infty \text{ then } \lim_{x \rightarrow c} f(x) = +\infty$$

$$\text{If } \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = -\infty \text{ then } \lim_{x \rightarrow c} f(x) = -\infty$$

$$\text{If } \lim_{x \rightarrow c^-} f(x) \neq \lim_{x \rightarrow c^+} f(x) \text{ then } \lim_{x \rightarrow c} f(x) = \text{DNE}$$

1.
$$\lim_{x \rightarrow 5^+} \frac{1}{x-5}$$

2.
$$\lim_{x \rightarrow 5^-} \frac{1}{x-5}$$

3.
$$\lim_{x \rightarrow 5} \frac{1}{x-5}$$

4.
$$\lim_{x \rightarrow 5^+} \frac{1}{(x-5)^2}$$

5.
$$\lim_{x \rightarrow 5^-} \frac{1}{(x-5)^2}$$

6.
$$\lim_{x \rightarrow 5} \frac{1}{(x-5)^2}$$

7.
$$\lim_{x \rightarrow 2^+} \frac{-3x}{x-2}$$

8.
$$\lim_{x \rightarrow 2^-} \frac{-3x}{x-2}$$

9.
$$\lim_{x \rightarrow 2} \frac{-3x}{x-2}$$

10.
$$\lim_{x \rightarrow 1^+} \frac{x+2}{1-x}$$

11.
$$\lim_{x \rightarrow 1^-} \frac{x+2}{1-x}$$

12.
$$\lim_{x \rightarrow 1} \frac{x+2}{1-x}$$

13.
$$\lim_{x \rightarrow -4^+} \frac{x}{x+4}$$

14.
$$\lim_{x \rightarrow -4^-} \frac{x}{x+4}$$

15.
$$\lim_{x \rightarrow -4} \frac{x}{x+4}$$

16.
$$\lim_{x \rightarrow 2^+} \frac{x+2}{x^2-4}$$

17.
$$\lim_{x \rightarrow 3^+} \frac{4x^2}{9-x^2}$$

18.
$$\lim_{x \rightarrow 0^+} \frac{\sqrt{3+x^2}}{x}$$

$$19. \lim_{x \rightarrow 2^+} \frac{x^2 + x - 6}{x^2 - 4x + 4}$$

$$20. \lim_{x \rightarrow 2^-} \frac{x^2 + x - 6}{x^2 - 4x + 4}$$

$$21. \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4x + 4}$$

$$22. \lim_{x \rightarrow 0^-} \frac{4x}{x^2 - 3x}$$

$$23. \lim_{x \rightarrow -3^-} \frac{x^2 - 9}{x^2 + 6x + 9}$$

$$24. \lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 6x + 9}$$

$$25. \lim_{x \rightarrow -2^+} \frac{x - 2}{x^2 - 4}$$

$$26. \lim_{x \rightarrow -2^-} \frac{x - 2}{x^2 - 4}$$

$$27. \lim_{x \rightarrow -2} \frac{x - 2}{x^2 - 4}$$