

**Converting a Log Equation with One Log Expression On Each Side of the Equation  
into an Polynomial Equation**

$$\log_b x = \log_b y \text{ is equivalent to } x = y$$

**Solving Logarithmic Equations with One Log Expression On Each Side of the Equation  
by converting it into an Polynomial Equation**

**Example 1**

Solve for x:

$$\log_2(4x + 1) = \log_2(x + 7)$$

use:  $\log_b(x) = \log_b(x) \Rightarrow x = y$   
(there can only be ONE log expression  
on each side of the equation to use the rule)

$$4x + 1 = x + 7$$

$$3x = 6$$

$$x = 2$$

Check:  $x = 2$ 

$$\log_2[4(2) + 1] = \log_2[2 + 7]$$

$$\log_2(9) = \log_2(9) \text{ Yes}$$

Solution:  $x = 2$ **Example 2**

Solve for x:

$$\log_5(2x - 5) = \log_5(-x + 13)$$

use:  $\log_b(x) = \log_b(x) \Rightarrow x = y$   
(there can only be ONE log expression  
on each side of the equation to use the rule)

$$2x - 5 = -x + 13$$

$$3x = 18$$

$$x = 6$$

Check:  $x = 6$ 

$$\log_5[2(6) - 5] = \log_5[-(6) + 13]$$

$$\log_2(7) = \log_2(7) \text{ Yes}$$

Solution:  $x = 6$

### Example 3

Solve for  $x$ :

$$\log_2(2x + 1) = \log_2(x^2 - 5x + 7)$$

use:  $\log_b(x) = \log_b(x) \Rightarrow x = y$   
(there can only be ONE log expression  
on each side of the equation to use the rule)

$$2x + 1 = x^2 - 5x + 7$$

$$0 = x^2 - 7x + 6$$

$$0 = (x - 6)(x - 1)$$

$$x = 6 \text{ or } x = 1$$

Check:  $x = 6$

$$\log_2(2x + 1) = \log_2(x^2 - 5x + 7)$$

$$\log_2[2(6) + 1] = \log_2[(6)^2 - 5(6) + 7]$$

$$\log_2(12 + 1) = \log_2(36 - 30 + 7)$$

$$\log_2(13) = \log_2(13) \text{ Yes}$$

Check:  $x = 1$

$$\log_2(2x + 1) = \log_2(x^2 - 5x + 7)$$

$$\log_2[2(1) + 1] = \log_2[(1)^2 - 5(1) + 7]$$

$$\log_2(2 + 1) = \log_2(1 - 5 + 7)$$

$$\log_2(3) = \log_2(3) \text{ Yes}$$

Solution(s):  $x = 6$  or  $x = 1$

#### Example 4

Solve for x:

$$\log_5(x) + \log_5(x - 4) = \log_5(x + 14) \quad \text{use: } \log_b(x) + \log_b(y) = \log_b(x \cdot y)$$

$$\log_5(x^2 - 4x) = \log_5(x + 14)$$

use:  $\log_b(x) = \log_b(x) \Rightarrow x = y$   
(there can only be ONE log expression  
on each side of the equation to use the rule)

$$x^2 - 4x = x + 14$$

$$x^2 - 5x - 14 = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = 7 \text{ or } x = -2$$

Check:  $x = 7$

$$\log_5(x) + \log_5(x - 4) = \log_5(x + 14)$$

$$\log_5(7) + \log_5(7 - 4) = \log_5(7 + 14)$$

$$\log_5(7) + \log_5(3) = \log_5(21)$$

$$\log_5(7 \cdot 3) = \log_5(21) \quad \text{Yes}$$

Check:  $x = -2$

$$\log_5(x) + \log_5(x - 4) = \log_5(x + 14)$$

$$\log_5(-2) + \log_5(-2 - 4) = \log_5(-2 + 14)$$

Stop: log (negative number)

$$x \neq -2$$

Solution(s):  $x = 7$

### Example 5

Solve for x:

$$\log_7(5x + 2) - \log_7(x - 2) = 2\log_7 3 \quad \text{use: } \log_b(x)^a = a \cdot \log_b(x)$$

$$\log_7(5x) - \log_7(x - 2) = \log_7 3^2 \quad \text{use: } \log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

$$\log_7\left(\frac{5x + 2}{x - 2}\right) = \log_7(9)$$

use:  $\log_b(x) = \log_b(x) \Rightarrow x = y$   
(there can only be ONE log expression  
on each side of the equation to use the rule)

$$\frac{5x + 2}{x - 2} = 9 \quad \text{note: multiply both sides by } x - 2$$

$$5x - 2 = 9x - 18$$

$$20 = 4x$$

$$5 = x$$

Check:  $x = 5$

$$\log_7[5(5) + 2] - \log_7[5 - 2] = 2\log_7 3$$

$$\log_7(25 + 2) - \log_7(3) = \log_7 3^2$$

$$\log_7(27) - \log_7(3) = \log_7(9)$$

$$\log_7\left(\frac{27}{3}\right) = \log_7(9)$$

$$\log_7(9) = \log_7(9) \quad \text{Yes}$$

Solution:  $x = 5$