Section 9 – 6A: Solving Logarithmic Equations

Converting a Log Equation with ONE Log Expression into an Exponential Equation

$$\log_b x = a \quad \text{is equivalent to} \quad x = b^a$$

Solving Logarithmic Equations with ONE log expression in the equation by converting a Log Equation with ONE log expression into an Exponential Equation

Example 1

Solve for x: $\log_4(5x-4)=2$

use: $\log_b x = a \;\Rightarrow\; x = b^a$
(the base of the log becomes the base under a as an exponent to convert the log equation into an exponential equation

$5x-4=4^2$

$5x-4=16$

$5x=20$
$x=4$

Check: $x=4$
$\log_4[5(4)-4]=2$
$\log_4(20-4)=2$
$\log_4(16)=2$
$16=4^2$

Solution: $x=4$

Example 2

Solve for x: $\log_2(4x+12)=3$

use: $\log_b x = a \;\Rightarrow\; x = b^a$
(the base of the log becomes the base under a as an exponent to convert the log equation into an exponential equation

$4x+12=2^3$

$4x+12=8$

$4x=-4$
$x=-1$

Check: $x=-1$
$\log_2[4(-1)+12]=3$
$\log_2(-4+12)=3$
$\log_2(8)=3$
$8=2^3$ Yes

Solution: $x=-1$

Example 3

Solve for x: $\frac{1}{2}\log_3(7x-5)=1$

use: $\mathrm{a}\log_b \mathrm{x}=\log_b(\mathrm{x})^{\mathrm{a}}$

$\log_3\sqrt{7x-5}=1$

use: $\log_b \mathrm{x}=\mathrm{a} \;\Rightarrow\; \mathrm{x}=\mathrm{b}^{\mathrm{a}}$
(the base of the log becomes the
base under a as an exponent
to convert the log equation
into an exponential equation

$\sqrt{7x-5}=3^1$

$\sqrt{7x-5}=3$ Note: square both sides

$7x-5=9$

$7x=14$

$x=2$

Check: $x=2$
$\frac{1}{2}\log_3\left[7(2)-5\right]=1$
$\frac{1}{2}\log_3(9)=1$
$\log_3\sqrt{9}=1$
$\log_3 3=1$
$3=3^1$ Yes

Solution: $x=2$

Example 3A

Solve for x: $\frac{1}{2}\log_3(7x-5)=1$

multiply both sides by 2

$\log_3(7x-5)=2$

use: $\log_b \mathrm{x}=\mathrm{a} \;\Rightarrow\; \mathrm{x}=\mathrm{b}^{\mathrm{a}}$
(the base of the log becomes the
base under a as an exponent
to convert the log equation
into an exponential equation

$7x-5=3^2$

$7x-5=9$

$7x=14$

$x=2$

Check: $x=2$
$\frac{1}{2}\log_3\left[7(2)-5\right]=1$
$\frac{1}{2}\log_3(9)=1$
$\log_3\sqrt{9}=1$
$\log_3 3=1$
$3=3^1$ Yes

Solution: $x=2$

Example 4

Solve for x:

$\log(x^2 - x - 19) = 0$ Note: Base 10

$\log_{10}(x^2 - x - 19) = 0$ use: $\log_b x = a \Rightarrow x = b^a$

$x^2 - x - 19 = 10^0$

$x^2 - x - 19 = 1$

$x^2 - x - 20 = 0$

$(x - 5)\ (x + 4) = 0$

$x = 5$ or $x = -4$

Check: $x = 5$

$\log_{10}\left[(5)^2 - (5) - 19\right] = 0$

$\log_{10}(25 - 5 - 19) = 0$

$\log_{10}(25 - 5 - 19) = 0$

$\log_{10}(1) = 0$

$1 = 10^0$ Yes

Check: $x = -4$

$\log_{10}\left[(-4)^2 - (-4) - 19\right] = 0$

$\log_{10}(16 + 4 - 19) = 0$

$\log_{10}(25 - 5 - 19) = 0$

$\log_{10}(1) = 0$

$1 = 10^0$ Yes

Solution(s): $x = 5$ or $x = -4$

Example 5

Solve for x:

$\log_3(x+6)+\log_3 x = 3$ use: $\log_b(x)+\log_b(y) = \log_b(x \bullet y)$

$\log_3\left[(x+6)\bullet x\right] = 3$

$\log_3\left[x^2+6x\right] = 3$ use: $\log_b \mathrm{x} = \mathrm{a} \Rightarrow \mathrm{x} = \mathrm{b}^{\mathrm{a}}$

$x^2+6x = 3^3$

$x^2+6x = 27$

$x^2+6x-27 = 0$

$(x+9)\ (x-3) = 0$

$x = -9$ or $x = 3$

Check: $x = -9$

$\log_3(-9+6)+\log_4(-9) = 3$

STOP: log(a negitive number)

$\mathrm{x} \neq -9$

Check: $x = 3$

$\log_3(3+6)+\log_3(3) = 3$

$\log_3(9)+\log_3(3) = 3$

$\log_3(27) = 3$

$27 = 3^3$

$27 = 27$ Yes

Solution(s): $x = 3$

Example 6

Solve for x:

$\log_2(5x-6) - \log_2(x) = 3$ use: $\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$

$\log_2\left(\frac{5x-6}{x}\right) = 3$ use: $\log_b x = a \Rightarrow x = b^a$

$\left(\frac{5x-6}{x}\right) = 2^3$

$\left(\frac{5x-6}{x}\right) = 8$ (multily both sides by x)

$5x - 6 = 8x$

$-6 = 3x$

$x = -2$

Check: $x = -2$

$\log_2[5(-2) - 6] - \log_2(-3) = 3$

$\log_2[-16] - \log_2(-2) = 3$

STOP: log(a negitive number)
$x \neq -2$

No Solution

Example 7

Solve for x:

$\log_4(2x) - \log_4(x-3) = 1$ use: $\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$

$\log_4\left(\frac{2x}{x-3}\right) = 1$ use: $\log_b x = a \Rightarrow x = b^a$

$\left(\frac{2x}{x-3}\right) = 4^1$

$\left(\frac{2x}{x-3}\right) = 4$ (multily both sides by $x - 3$)

$2x = 4x - 12$

$-2x = -12$

$x = 6$

Check: $x = 6$

$\log_4(2x) - \log_4(x-3) = 1$

$\log_4[2(6)] - \log_4[6-3)] = 1$

$\log_4(12) - \log_4(3) = 1$

$\log_4\left(\frac{12}{3}\right) = 1$

$\log_4(4) = 1$

$4 = 4^1$ Yes

Solution: $x = 6$

