

Section 9 – 5B: Properties of Logarithms

Express as the sum, difference and/or product

Use the following Property of Logarithmic Functions

$$\log_b(x) + \log_b(y) = \log_b(x \cdot y)$$

to express the **sum** of logarithms as **single log function**

Example 1

$$\log_b(x) + \log_b(x + 1)$$

can be expressed as
the single log function

$$\log_b[(x) \cdot (x + 1)]$$

$$\log_b(x^2 + x)$$

Example 2

$$\log_b(4x) + \log_b(x + 3)$$

can be expressed as
the single log function

$$\log_b[4x \cdot (x + 3)]$$

$$\log_b(4x^2 + 12x)$$

Use the following Property of Logarithmic Functions

$$\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

to express the **difference** of logarithms as a **single log function**

Example 3

$$\log_b(x - 3) - \log_b(x + 4)$$

can be expressed as
the single log function

$$\log_b\left(\frac{x - 3}{x + 4}\right)$$

Example 4

$$\log_b(\sqrt{x - 2}) - \log_b(5)$$

can be expressed as
the single log function

$$\log_b\left(\frac{\sqrt{x - 2}}{5}\right)$$

Use the following Property of Logarithmic Functions

$$a \cdot \log_b(x) = \log_b(x)^a$$

to express the **product** of logarithms as a single log function

Example 5

$$4 \log_2(2x - 3)$$

can be expressed as
the single log function

$$= \log_2(2x - 3)^4$$

Example 6

$$\frac{1}{2} \cdot \log_6(x - 2)$$

can be expressed as
the single log function

$$= \log_6(x - 2)^{1/2}$$

$$\log_6 \sqrt{x - 2}$$

Use the Properties of Logarithmic Functions to express the sum, difference and/or products of logarithms as a **single log function**

Example 10

$$3 \log_5(4) + 2 \log_5(x)$$

use: $a \cdot \log_b(x) = \log_b(x)^a$

$$\log_5(4^3) + \log_5(x^2)$$

use: $\log_b(x) + \log_b(y) = \log_b(x \cdot y)$

$$\log_5(64 \cdot x^2)$$

$$= \log_5(64x^2)$$

Example 11

$$2 \log x + \log(x - 3) \quad \text{use: } a \log_b(x) = \log_b(x)^a$$

$$\log(x^2) + \log(x - 3) \quad \text{use: } \log_b(x) + \log_b(y) = \log_b(x \cdot y)$$

$$= \log(x^3 - 3x^2)$$

Example 12

$$2 \log_b(5) - 3 \log_b(x - 2) \quad \text{use: } a \cdot \log_b(x) = \log_b(x)^a$$

$$\log_b(5^2) - \log_b(x - 2)^3$$

$$\log_b(25) - \log_b(x - 2)^3 \quad \text{use: } \log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

$$\log_b\left(\frac{25}{(x - 2)^3}\right)$$

Example 14

$$3 \log_b(2) + 3 \log_b(y + 4) - \frac{1}{2} \log_b(x - 9) \quad \text{use: } a \cdot \log_b(x) = \log_b(x)^a$$

$$\log_b(2^3) + \log_b(y + 4)^3 - \log_b(x - 9)^{1/2}$$

$$\log_b(8) + \log_b(y + 4)^3 - \log_b \sqrt{x - 9} \quad \text{use: } \log_b(x) + \log_b(y) = \log_b(x \cdot y)$$

$$\text{and } \log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

$$\log_b\left(\frac{8(y + 4)^3}{\sqrt{x - 9}}\right)$$