

Section 9 – 5A: Properties of Logarithms

Express as the sum, difference and/or product

Use the following Property of Logarithmic Functions

$$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

to express the single log function as the **sum** of logarithms

Example 1

$$\log_b(x \cdot y)$$

can be expressed as
the sum

$$= \log_b(x) + \log_b(y)$$

Example 2

$$\log_2(5 \cdot x)$$

can be expressed as
the sum

$$= \log_2(5) + \log_2(x)$$

Example 3

$$\log_b(2 \cdot x \cdot w)$$

can be expressed as
the sum

$$= \log_2(2) + \log_2(x) + \log_2(w)$$

Use the following Property of Logarithmic Functions

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

to express the single log function as the **difference** of logarithms

Example 4

$$\log_b\left(\frac{2}{x}\right)$$

can be expressed as the
difference

$$= \log_b(2) - \log_b(x)$$

Example 5

$$\log_5\left(\frac{y}{3}\right)$$

can be expressed as the
difference

$$= \log_5(y) - \log_5(3)$$

Example 6

$$\ln\left(\frac{x+1}{y}\right)$$

can be expressed as the
difference

$$= \ln(x+1) - \ln(y)$$

Use the following Property of Logarithmic Functions

$$\log_b(x)^a = a \cdot \log_b(x)$$

to express the single log function as the **product** of logarithms

Example 7

$$\log_b x^5$$

can be expressed as
the product

$$= 5 \cdot \log_b(x)$$

Example 8

$$\ln(x-1)^3$$

can be expressed as
the product

$$= 3 \cdot \ln(x-1)$$

Note: $\ln(x-1)$ is $\log_e(x-1)$

Example 9

$$\log(2x+3)^5$$

can be expressed as
the product

$$= 5 \cdot \log(2x+3)$$

Note: The base is 10

Use the Properties of Logarithmic Functions to express each single log function as the sum, difference and/or product

$$\log_3(x^2y^3)$$

$$\log_3(x^2) + \log_3(y^3)$$

$$= 2\log_3(x) + 3\log_3(y)$$

Example 10

use: $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$

use: $\log_b(x)^a = a \cdot \log_b(x)$

Example 11

$$\log_3(x^5\sqrt{y})$$

$$\log_3(x^2) + \log_3(y^{1/2})$$

$$= 5 \log_3(x) + \frac{1}{2} \log_3(y)$$

use: $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$

use: $\log_b(x)^a = a \cdot \log_b(x)$

Example 12

$$\log_b(81x^5) \quad \text{use: } \log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

$$\log_b(81) + \log_b(x^5) \quad \text{Note: } 81 = 3^4$$

$$\log_b(3^4) + \log_b(x^5) \quad \text{use: } \log_b(x)^a = a \cdot \log_b(x)$$

$$= 4 \log_b(3) + 5 \log_b(x)$$

Example 13

$$\log_b\left(\frac{x^2}{y^3}\right) \quad \text{use: } \log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(x^2) - \log_b(y^3) \quad \text{use: } \log_b(x)^a = a \cdot \log_b(x)$$

$$2\log_b(x) - 3\log_b(y)$$

Example 14

$$\log_b\left(\frac{25x^7}{\sqrt{y}}\right) \quad \text{use: } \log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(25x^7) - \log_b(\sqrt{y}) \quad \text{use: } \log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

$$\log_b(25) + \log_b(x^7) - \log_b(\sqrt{y}) \quad \text{Note: } 25 = 5^2 \quad \text{and} \quad \sqrt{y} = y^{1/2}$$

$$\log_b(5^2) + \log_b(x^7) - \log_b(y^{1/2}) \quad \text{use: } \log_b(x)^a = a \cdot \log_b(x)$$

$$= 2\log_b(5) + 7\log_b(x) - \frac{1}{2}\log_b(y)$$