

Section 9 – 4B: Graphs of Decreasing Logarithmic Functions

We want to determine what the graph of the logarithmic function

$$y = \log_a(x)$$

is for values of a such that $0 < a < 1$

We will select a value a such that $0 < a < 1$ and examine several ordered pairs for

$$y = \log_a(x)$$

We will use $a = 1/2$

The patterns we find for $a = 1/2$ will be true for any value of a if a is $0 < a < 1$

The graph of $y = \log_{(1/2)}(x)$

The logarithmic function $y = \log_{(1/2)}(x)$ can be written in exponential form as

$$x = (1/2)^y$$

x and y values for the left side of the graph

if $x = \frac{1}{2}$	if $x = \frac{1}{4}$	if $x = \frac{1}{8}$	if $x = \frac{1}{16}$	if $x = \frac{1}{32}$	if $x = \frac{1}{64}$
for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$
$\frac{1}{2} = \left(\frac{1}{2}\right)^y$	$\frac{1}{4} = \left(\frac{1}{2}\right)^y$	$\frac{1}{8} = \left(\frac{1}{2}\right)^y$	$\frac{1}{16} = \left(\frac{1}{2}\right)^y$	$\frac{1}{32} = \left(\frac{1}{2}\right)^y$	$\frac{1}{64} = \left(\frac{1}{2}\right)^y$
$y = 1$	$y = 2$	$y = 3$	$y = 4$	$y = 5$	$y = 6$

As the values for x get closer and closer to zero then the values of y become larger and larger positive values.

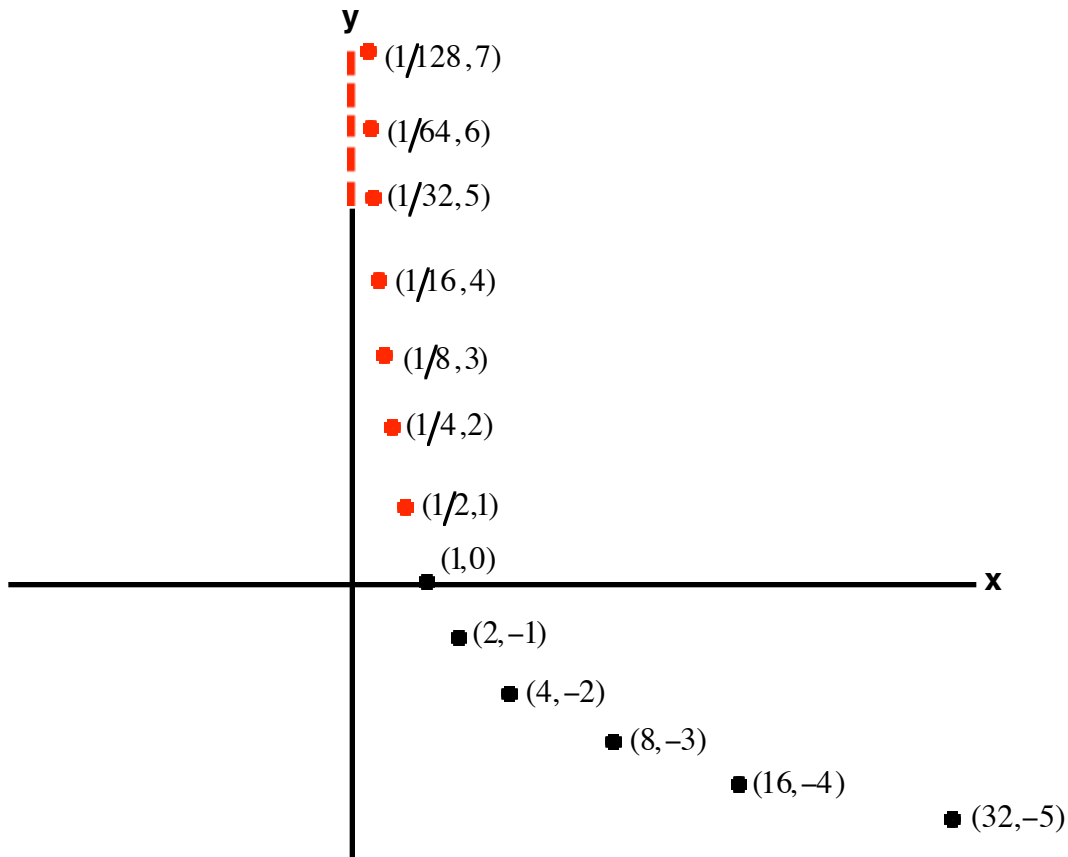
x and y values for the right side of the graph

if $x = 1$	if $x = 2$	if $x = 4$	if $x = 8$	if $x = 16$	if $x = 32$
for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$	for $x = \left(\frac{1}{2}\right)^y$
$1 = \left(\frac{1}{2}\right)^y$	$2 = \left(\frac{1}{2}\right)^y$	$4 = \left(\frac{1}{2}\right)^y$	$8 = \left(\frac{1}{2}\right)^y$	$16 = \left(\frac{1}{2}\right)^y$	$32 = \left(\frac{1}{2}\right)^y$
$y = 0$	$y = -1$	$y = -2$	$y = -3$	$y = -4$	$y = -5$

As the values for x become larger and larger positive numbers then the values for y become larger and larger negative values.

The table below shows several values for pairs of (x, y) $y = \log_{(1/2)}(x)$

x	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64	128	256
y	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8

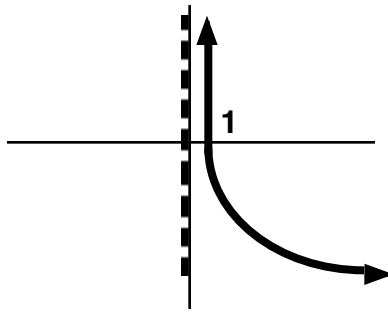


The graph of $y = \log_{(1/2)}(x)$

- Domain – Range:** The domain of the log function $y = \log_{(2)}(x)$ is $x > 0$. This means that the entire graph of the function will be to the right of the y axis. The range for y is all real numbers.
- The x intercept is at (1 , 0)
- Left End of the Graph:** As the values for x get closer and closer to zero then the values of y become larger and larger positive values. We show the graph of the left end of the graph pointing up and getting closer and closer to the y axis without touching the y axis. The Y axis is a vertical asymptote for the left side of the graph. We use a dotted line to show the asymptotic line. Every decreasing logarithmic function has a vertical asymptote.
- Right End of the Graph:** As the values for x become larger and larger positive numbers then the values for y become larger and larger negative values. We show the right end of the graph as a slowly decreasing curve pointing down and to the right and we use an arrow to show that it continues to decrease.

The Graph of an **Decreasing** Logarithmic Function

The graph of $y = \log_a(x)$ for all values of a such that $0 > a > 1$ is shown below.

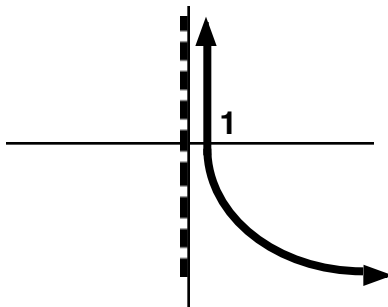


$y = \log_a(x)$ for all values of a such that $0 > a > 1$ is called a **decreasing logarithmic function**

1. **Domain – Range:** The domain of the log function $y = \log_{(2)}(x)$ is $x > 0$. This means that the entire graph of the function will be to the right of the y axis. The range for y is all real numbers.
2. The **y axis is a vertical asymptote**. We use a **dotted line** to show the asymptotic line. Every increasing logarithmic function has a vertical asymptote.
3. The **x Intercept:** Every graph of $y = \log_a(x)$ for all values of a such that $0 > a > 1$ will have an **x intercept** of $(1, 0)$
4. **Left End of the Graph:** As the values for x get closer and closer to zero then the values of y have larger and larger positive values. We show the graph of the left end of the graph pointing up and getting closer and closer to the y axis without touching the y axis. The **Y axis is a vertical asymptote for the left side of the graph**. We use a **dotted line** to show the asymptotic line. Every decreasing logarithmic function has a vertical asymptote.
5. **Right End of the Graph:** As the values for x get larger the y values then the values of y have larger and larger negative values. The x values get larger much faster than the y values do so the right end of the graph decreases less rapidly than the exponential graph did. We show the right end of the graph as a **slowly decreasing curve pointing down and to the right** and we use an arrow to show that it continues to decrease.

There are 6 different transformations that change the position of the graph, asymptote and x intercept. The graph can be moved RiGHT, moved LEFT, moved UP, moved Down, flipped about the x AXIS or flipped about the y AXIS. These transformations are caused by the addition, subtraction or multiplication of various part of the equation.

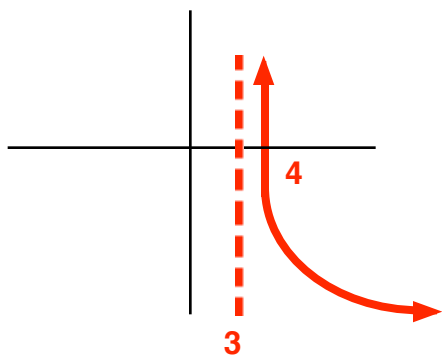
Translating the graph of $y = \log_{(1/2)}(x)$



Translation 1:

$$y = \log_{(1/2)}(x - 3)$$

subtracting 3 from x
moves the graph **RIGHT 3**



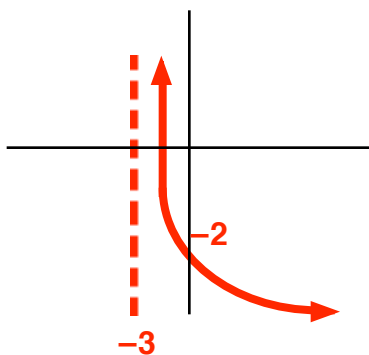
The x intercept is found by letting $y = 0$ and finding x

$$\begin{aligned} y &= \log_{(1/2)}(x - 3) \\ 0 &= \log_{(1/2)}(x - 3) \\ (1/2)^0 &= x - 3 \\ 1 &= x - 3 \\ 4 &= x \end{aligned}$$

Translation 2:

$$y = \log_{(1/2)}(x + 3)$$

adding 3 to x
moves the graph **LEFT 3**



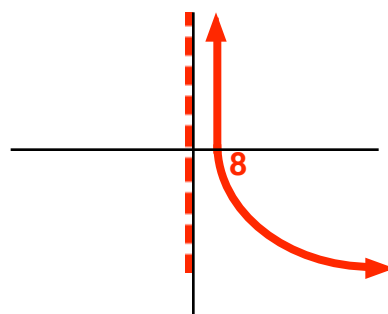
The x intercept is found by letting $y = 0$ and finding x

$$\begin{aligned} y &= \log_{(1/2)}(x + 3) \\ 0 &= \log_{(1/2)}(x + 3) \\ (1/2)^0 &= x + 3 \\ 1 &= x + 3 \\ -2 &= x \end{aligned}$$

Translation 3:

$$y = \log_{(1/2)}x + 3$$

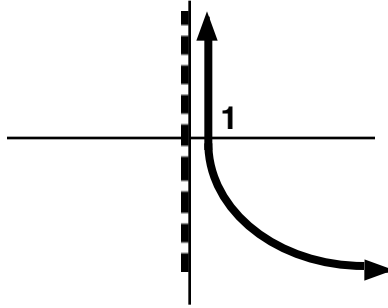
Adding 3 at the end
moves the graph **UP 3**



The x intercept is found by letting $y = 0$ and finding x

$$\begin{aligned} y &= \log_{(1/2)}x + 3 \\ 0 &= \log_{(1/2)}x + 3 \\ -3 &= \log_{(1/2)}x \\ (1/2)^{-3} &= x \\ 8 &= x \end{aligned}$$

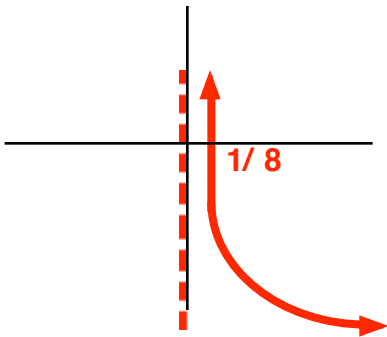
Translating the graph of $y = \log_{(1/2)}(x)$



Translation 4:

$$y = \log_{(1/2)} x - 3$$

subtracting 3 at the end
moves the graph **DOWN 3**



The x intercept is found by
letting $y = 0$ and finding x

$$y = \log_{(1/2)} x - 3$$

$$0 = \log_{(1/2)} x - 3$$

$$3 = \log_{(1/2)} x$$

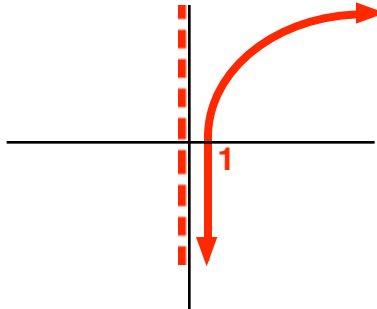
$$\left(\frac{1}{2}\right)^3 = x$$

$$\frac{1}{8} = x$$

Translation 5:

$$y = -\log_{(1/2)} x$$

negating y **flips the graph**
about the x axis



The x intercept is found by
letting $y = 0$ and finding x

$$y = -\log_{(1/2)} x$$

$$0 = -\log_{(1/2)} x$$

$$0 = \log_{(1/2)} x$$

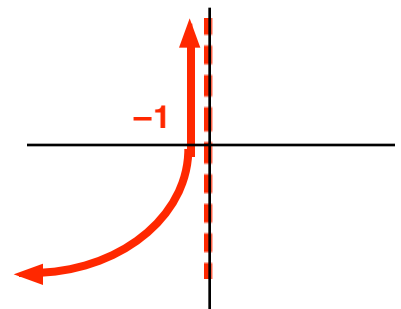
$$\left(\frac{1}{2}\right)^0 = x$$

$$1 = x$$

Translation 6:

$$y = \log_{(1/2)} (-x)$$

negating x **flips the graph**
about the y axis



The x intercept is found by
letting $y = 0$ and finding x

$$y = \log_{(1/2)} (-x)$$

$$0 = \log_{(1/2)} (-x)$$

$$\left(\frac{1}{2}\right)^0 = -x$$

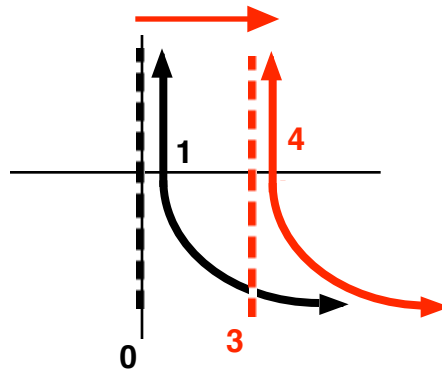
$$1 = -x$$

$$-1 = x$$

The graph of $y = \log_{(1/2)}(x - 3)$ compared to $y = \log_{(1/2)}(x)$

Subtracting 3 from the x inside a bracket moves the graph 3 units to the **RIGHT**

Compared to $y = \log_{(1/2)}(x)$ the graph of $y = \log_{(1/2)}(x - 3)$ moves 3 to the right



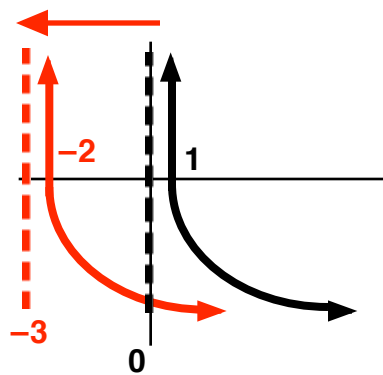
$y = \log_{(1/2)}(x)$ has an x intercept of 1

$y = \log_{(1/2)}(x - 3)$ has an x intercept of 4

The graph of $y = \log_{(1/2)}(x + 3)$ compared to $y = \log_{(1/2)}(x)$

Adding 3 to the x inside a bracket moves the graph 3 units to the **LEFT**

The graph of $y = \log_{(1/2)}(x + 3)$ moves 3 to the left compared to $y = \log_{(1/2)}(x)$ t

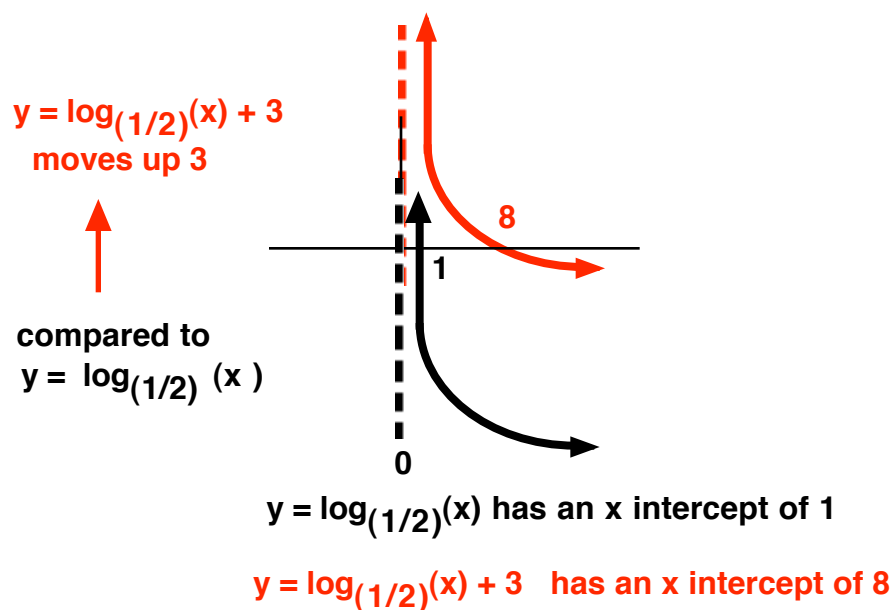


$y = \log_{(1/2)}(x)$ has an x intercept of 1

$y = \log_{(1/2)}(x + 3)$ has an x intercept of -2

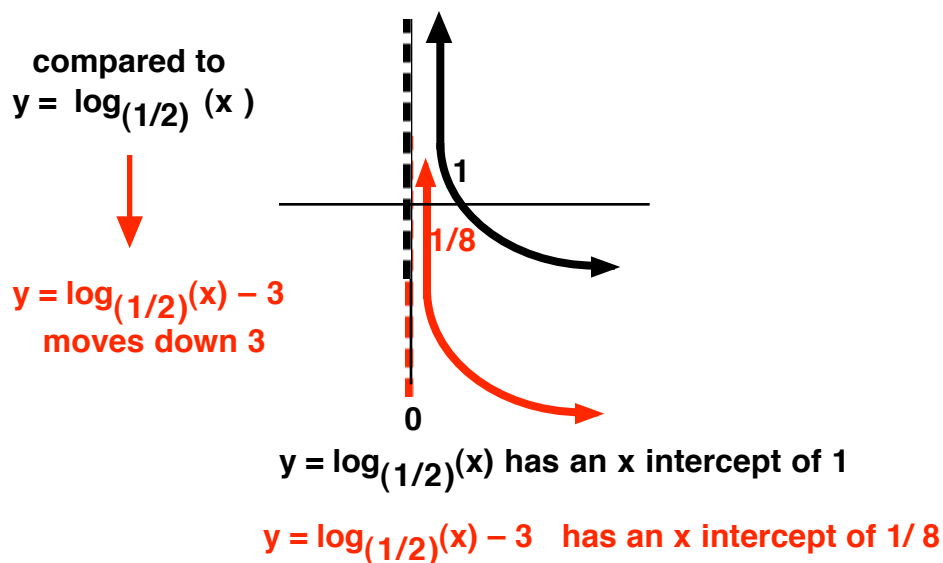
The graph of $y = \log_{(1/2)}(x) + 3$ compared to $y = \log_{(1/2)}(x)$

Adding 3 to the $\log_{(1/2)}(x)$ at the end of the equation moves the graph **up 3 units**.

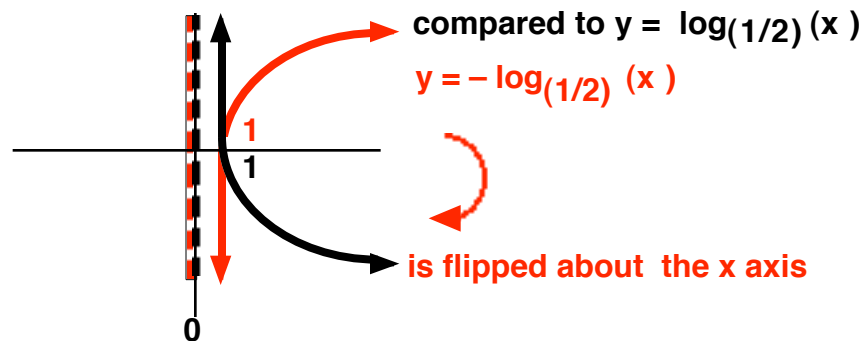


The graph of $y = \log_{(1/2)}(x) - 3$ compared to $y = \log_{(1/2)}(x)$

Subtracting 3 from the $\log_{(1/2)}(x)$ at the end of the equation moves the graph **DOWN 3 units**



The graph of $y = -\log_{(1/2)} x$ compared to $y = \log_{(1/2)}(x)$
 (negating the y values flips the graph about the x axis)

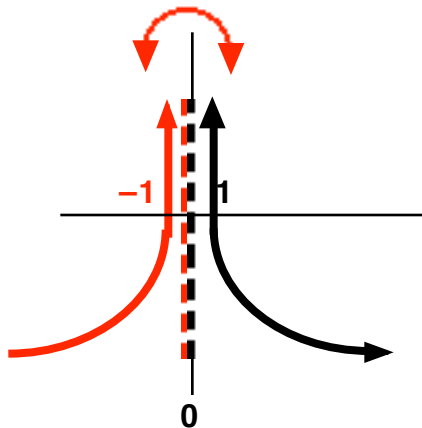


$y = \log_{(1/2)}(x)$ has an x intercept of 1

$y = -\log_{(1/2)}(x)$ has an x intercept of 1

The graph of $y = \log_{(1/2)}(-x)$ compared to $y = \log_{(1/2)}(x)$
 (negating the x values flips the graph about the y axis)

$y = \log_{(1/2)}(-x)$ is flipped about the y axis compared to $y = \log_2(x)$



$y = \log_{(1/2)}(x)$ has an x intercept of 1

$y = \log_{(1/2)}(-x)$ has an x intercept of -1