Section 9 – 4A: Graphs of Increasing Logarithmic Functions

We want to determine what the graph of the logarithmic function

$y = \log_a(x)$

looks like for values of a such that a > 1

We will select a value a such that a > 1 and examine several ordered pairs for

$y = \log_a(x)$

We will use a = 2

The patterns we find for a = 2 will be true for any value of a if a is > 1

The graph of $y = \log_2(x)$

The logarithmic function $y = \log_2(x)$ can be written in exponential form as

$x = 2^y$

x and y values for the right side of the graph

if $x = 2$	if $x = 4$	if $x = 8$	if $x = 16$	if $x = 32$	if $x = 64$
for $x = 2^y$	for $x = 2^y$	for $x = 2^y$	for $x = 2^y$	for $x = 2^y$	for $x = 2^y$
$2 = 2^y$	$4 = 2^y$	$8 = 2^y$	$16 = 2^y$	$32 = 2^y$	$64 = 2^y$
$y = 1$	$y = 2$	$y = 3$	$y = 4$	$y = 5$	$y = 6$

As the values for **x become larger and larger positive numbers**
then the values for **y become larger and larger positive values.**

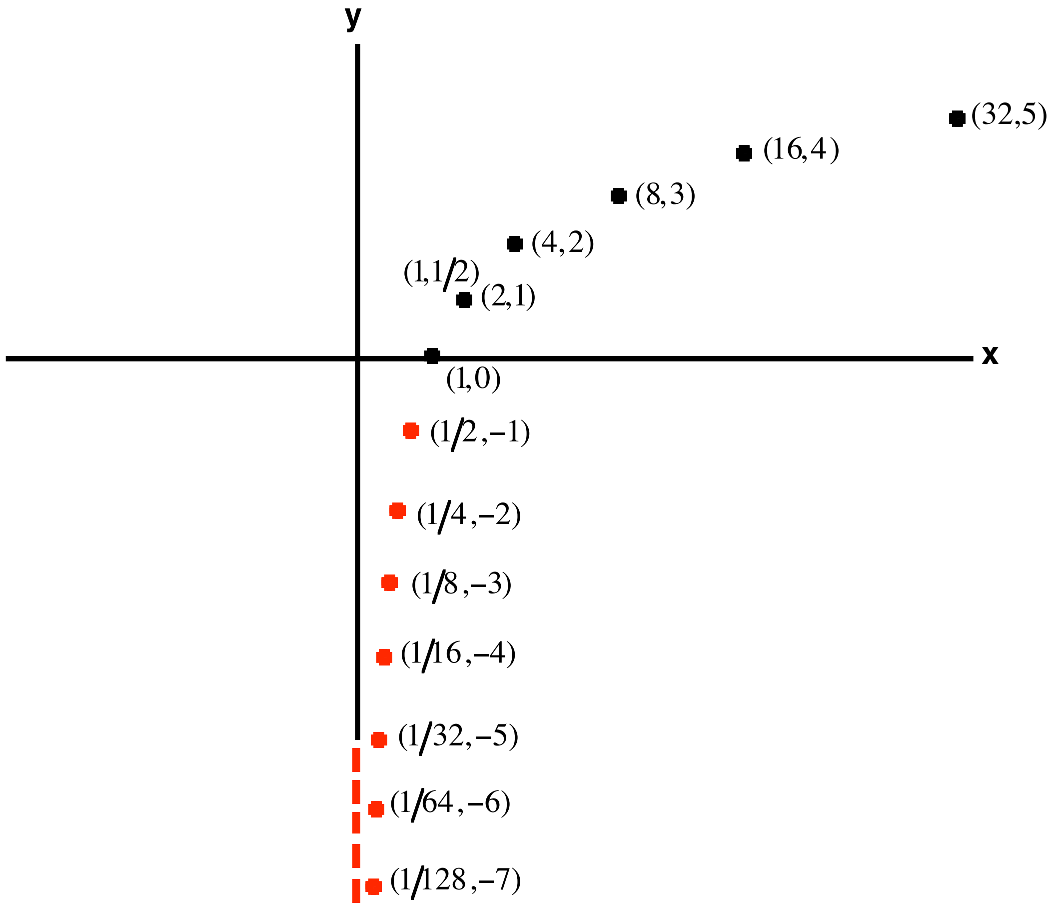
x and y values for the left side of the graph

if $x = \frac{1}{64}$	if $x = \frac{1}{32}$	if $x = \frac{1}{16}$	if $x = \frac{1}{8}$	if $x = \frac{1}{8}$	if $x = \frac{1}{2}$
for $x = 2^y$	for $x = 2^y$	for $x = 2^y$	for $x = 2^y$	for $x = 2^y$	for $x = 2^y$
$\frac{1}{64} = 2^y$	$\frac{1}{32} = 2^y$	$\frac{1}{16} = 2^y$	$\frac{1}{8} = 2^y$	$\frac{1}{8} = 2^y$	$\frac{1}{2} = 2^y$
$y = -6$	$y = -5$	$y = -4$	$y = -3$	$y = -3$	$y = -1$

As the values for **x get closer and closer to zero**
then the values of **y become larger and larger negative values.**

The table below shows several values for pairs of (x, y) for **y = 2^x**

x	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64	128	256
y	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8



The graph of $y = \log_{(2)}(x)$

1. **Domain – Range: The domain of the log function $y = \log_{(2)}(x)$ is $x > 0$** This means that the entire graph of the function will be to the right of the y axis. The range for y is all real numbers.

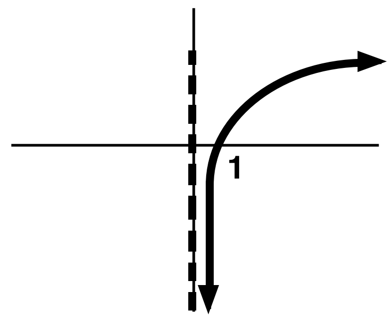
2. The **x intercept is at (1 , 0)**

3. **Left end of the graph**: As the values for **x get closer and closer to zero** then the values of **y become larger and larger negative values.** We show the graph of the left end of the graph pointing down and getting closer and closer to the y axis without touching the y axis. The **Y axis is a vertical asymptote for the left side of the graph**. We use a **dotted line** to show the asymptotic line. Every decreasing logarithmic function has a vertical asymptote.

4. **Right end of the graph**: As the values for **x become larger and larger positive numbers** then the values for **y become larger and larger positive values.** We show the right end of the graph as **a slowly increasing curve pointing up and to the right** and we use an arrow to show that it continues to increase.

The Graph of an Increasing Logarithmic Function

The graph of **$y = \log_a (x)$ for all values of a such that $a > 1$** is shown below.



$y = \log_a (x)$ for all values of a such that $a > 1$ is called a **increasing logarithmic function**

1. **Domain – Range:** x can only be **positive numbers greater than 0. The domain is $x > 0$** This means that the entire graph of the function will be to the right of the y axis. The range for y is **is all real numbers.**

2. The **y axis is a vertical asymptote**. We use a **dotted line** to show the asymptotic line. Every increasing logarithmic function has a vertical asymptote.

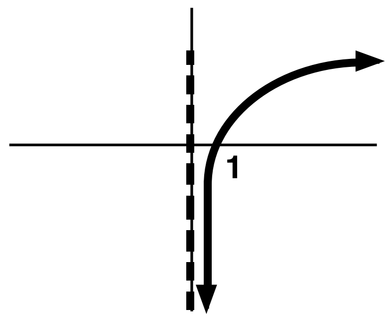
3. The **x Intercept:** Every graph of **$y = \log_a (x)$ for all values of a such that $a > 1$** will have an **x intercept** of (1, 0)

5. **Left end of the Graph**: As the values for x get closer and closer to zero then the values of y have larger and larger negative values. We show the graph of the left end of the graph pointing down and getting closer and closer to the y axis without touching the y axis. The **y axis is a vertical asymptote for the left side of the graph**. We use a **dotted line** to show the asymptotic line. Every increasing logarithmic function has a vertical asymptote.

4. **Right end of the Graph:** As the values for x get larger the y values then the values of y have larger and larger positive values. The x values get larger much faster than the y values do so the right end of the graph decreases less rapidly than the exponential graph did. We show the right end of the graph as **a slowly increasing curve pointing up and to the right** and we use an arrow to show that it continues to decrease.

There are 6 different transformation that change the position of the graph, asymptote and x intercept. The graph of $y = \log_2 x$ and be moved RiGHT, moved LEFT, moved UP, moved Down, flipped about the X AXIS or flipped about the Y AXIS. These transformations are effected by the addition, subtraction or multiplication of various part of the equation.

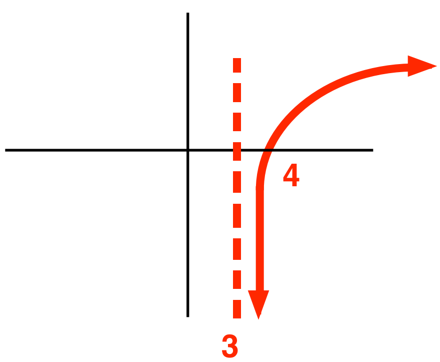
Translating the graph of $y = \log_2 (x)$



Translation 1:

$y = \log_2 (x - 3)$

subtracting 3 from x
moves the graph **RIGHT 3**



The x intercept is found by letting y = 0 and finding x

$y = \log_2 (x - 3)$

$0 = \log_2 (x - 3)$

$2^0 = x - 3$

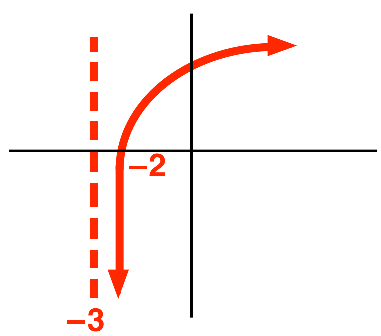
$1 = x - 3$

$4 = x$

Translation 2:

$y = \log_2 (x + 3)$

adding 3 to x
moves the graph **LEFT 3**



The x intercept is found by letting y = 0 and finding x

$y = \log_2 (x + 3)$

$0 = \log_2 (x + 3)$

$2^0 = x + 3$

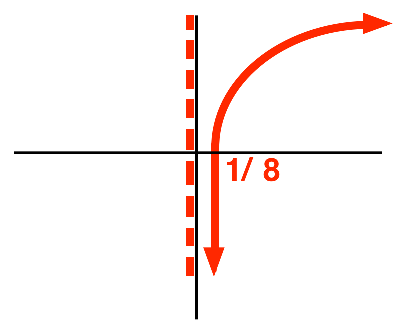
$1 = x + 3$

$-2 = x$

Translation 3:

$y = \log_2(x) + 3$

adding 3 at the end
moves the graph **UP 3**



The x intercept is found by letting y = 0 and finding x

$y = \log_2(x) + 3$

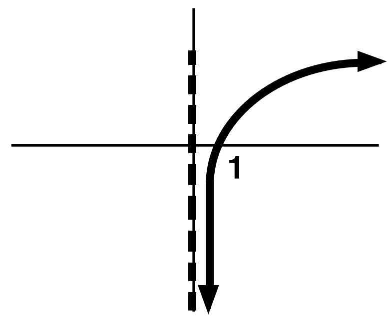
$0 = \log_2(x) + 3$

$-3 = \log_2(x)$

$2^{-3} = x$

$1/8 = x$

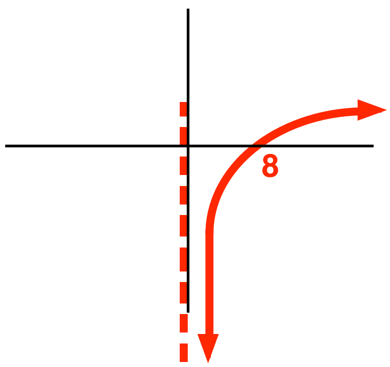
Translating the graph of $y = \log_2(x)$



Translation 4:

$y = \log_2(x) - 3$

subtracting 3 at the end moves the graph **DOWN 3**



The x intercept is found by letting y = 0 and finding x

$y = \log_2(x) - 3$

$0 = \log_2(x) - 3$

$3 = \log_2(x)$

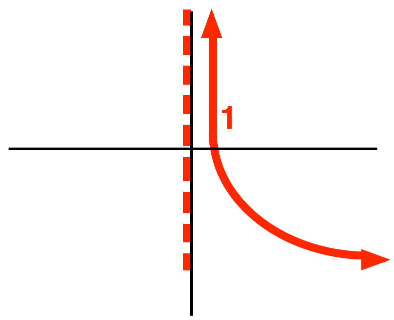
$2^3 = x$

$8 = x$

Translation 5:

$y = -\log_2(x)$

negating y **flips the graph about the x axis**



The x intercept is found by letting y = 0 and finding x

$y = -\log_2(x)$

$0 = -\log_2(x)$

$0 = \log_2(x)$

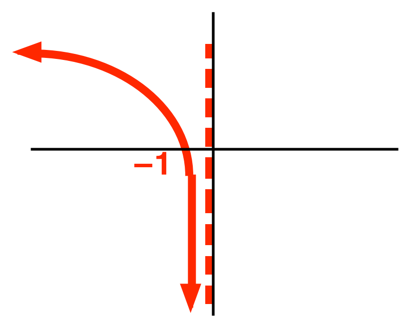
$2^0 = x$

$1 = x$

Translation 6:

$y = \log_2(-x)$

negating x **flips the graph about the y axis**



The x intercept is found by letting y = 0 and finding x

$y = \log_2(-x)$

$0 = \log_2(-x)$

$2^0 = -x$

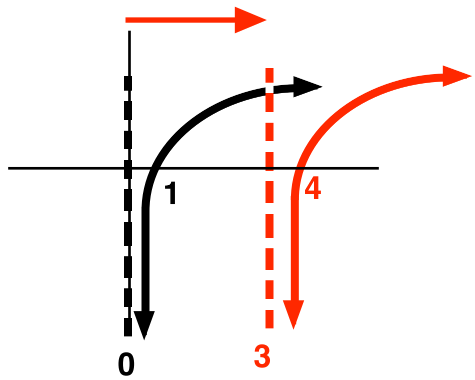
$1 = -x$

$-1 = x$

The graph of $y = \log_2(x - 3)$ compared to the graph of $y = \log_2(x)$

Subtracting 3 from the x inside a bracket moves the graph 3 units to the RIGHT

Compared to $y = \log_2(x)$ the graph of $y = \log_2(x - 3)$ moves 3 to the right



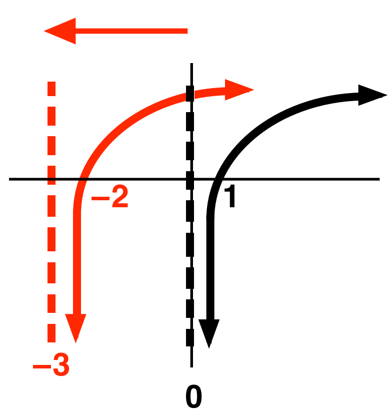
$y = \log_2(x)$ has a y intercept of 1

$y = \log_2(x - 3)$ has a y intercept of 4

The graph of $y = \log_2(x + 3)$ compared to the graph $y = \log_2(x)$

Adding 3 to the x inside a bracket moves the graph 3 units to the LEFT

The graph of $y = \log_2(x + 3)$ moves 3 to the left compared to $y = \log_2(x)$

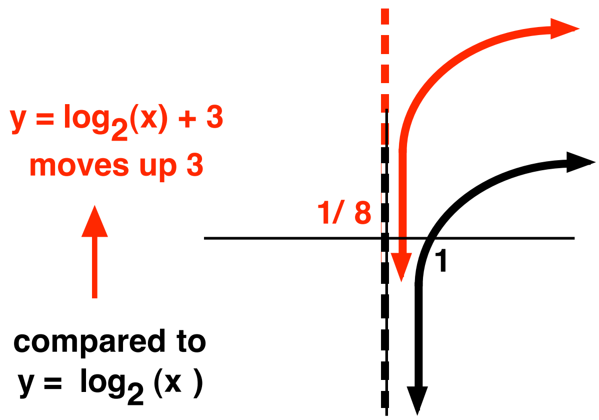


$y = \log_2(x)$ has a y intercept of 1

$y = \log_2(x + 3)$ has a y intercept of –2

The graph of $y = \log_2(x) + 3$ compared to the graph $y = \log_2(x)$

Adding 3 to the $\log_2(x)$ at the end of the equation moves the graph UP 3 units

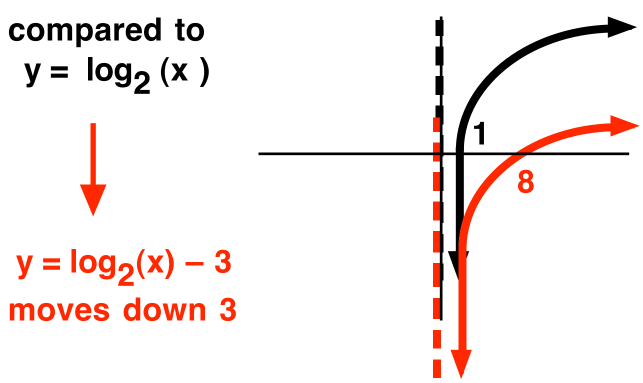


$y = \log_2(x)$ has a y intercept of 1

$y = \log_2(x) + 3$ has a y intercept of 1/ 8

The graph of $y = \log_2(x) - 3$ compared to the graph of $y = \log_2(x)$

Subtracting 3 from the $\log_2(x)$ at the end of the equation moves the graph DOWN 3 units

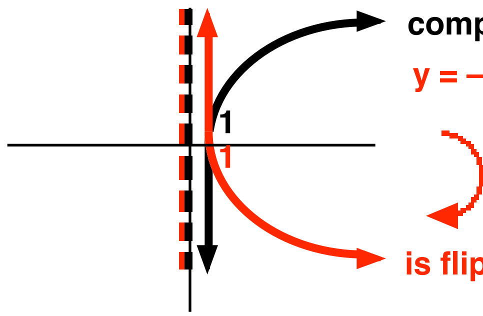


$y = \log_2(x)$ has a y intercept of 1

$y = \log_2(x) - 3$ has a y intercept of 8

The graph of $y = -\log_2(x)$ compared to $y = \log_2(x)$

(**negating the y values flips the graph about the x axis**)



compared to $y = \log_2(x)$

$y = -\log_2(x)$

is flipped about the x axis

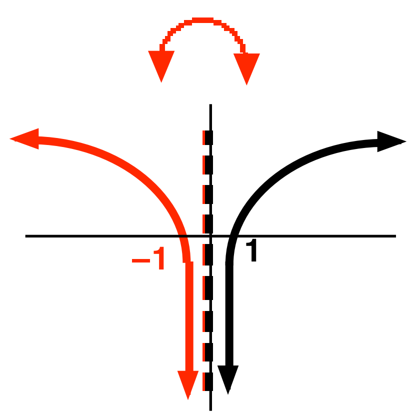
$y = \log_2(x)$ has a y intercept of 1

$y = -\log_2(x)$ has a y intercept of 1

The graph of $y = \log_2(-x)$ compared to the graph of $y = \log_2(x)$

(**negating the x values flips the graph about the y axis**)

$y = \log_2(-x)$ is flipped about the y axis compared to $y = \log_2(x)$



$y = \log_2(x)$ has a y intercept of 1

$y = \log_2(-x)$ has a y intercept of 1