9 – 3C: Evaluating Basic Logarithmic Expressions

Evaluating a Logaritmatic Expression involves finding the value of the expression. If you do not know the value of the expression you can set it equal to a variable (we will chose x) to create an equation. You can then solve the equation as we did in the problems in the last section.

Example 1

Evaluate: $\log_4 16$

set the log expression equal
to x and solve for x

$\log_4 16 = x$
$16 = 4^x$
$4^2 = 4^x$
$x = 2$
so $\log_4 16 = 2$

Example 2

Evaluate: $\log_2 32$

set the log expression equal
to x and solve for x

$\log_2 32 = x$
$32 = 2^x$
$2^5 = 2^x$
$x = 5$
so $\log_2 32 = 5$

Example 3

Evaluate: log 1000
no base is shown so
it is base 10

set the log expression equal
to x and solve for x

$\log_{10} 1000 = x$
$1000 = 10^x$
$10^3 = 10^x$
$x = 3$
so $\log_{10} 1000 = 3$

Example 4

Evaluate: log .01
no base is shown so
it is base 10

set the log expression equal
to x and solve for x

$\log_{10} .01 = x$
$\frac{1}{100} = 10^x$
$10^{-2} = 10^x$
$x = -2$
so $\log_{10} .01 = -2$

Example 5

Evaluate: $\log_8 2$

set the log expression equal
to x and solve for x

$\log_8 2 = x$

$2 = 8^x$

$2 = \left(2^3\right)^x$

$2^1 = 2^{3x}$

$1 = 3x$

$x = 1/3$

so $\log_8 2 = 1/3$

Example 6

Evaluate: $\log_{81} 3$

set the log expression equal
to x and solve for x

$\log_{81} 3 = x$

$3 = 81^x$

$3 = \left(3^4\right)^x$

$3^1 = 3^{4x}$

$1 = 4x$

$x = 1/4$

so $\log_{81} 3 = 1/4$

Example 7

Evaluate: $\ln e^5$

ln is the natural log
it has base e

set the log expression equal
to x and solve for x

$\ln_e e^5 = x$

$e^5 = e^x$

$x = 5$

so $\ln e^5 = 5$

Example 8

Evaluate: $\log_5 1$

set the log expression equal
to x and solve for x

$\log_5 1 = x$

$1 = 5^x$

$1 = 5^0 = 0^x$

$x = 0$

so $\log_5 1 = 0$

Alternate Solution Technique

If you understand what question is being asked then you may be able to evaluate an expression by thinking of the number that answers the question being asked. Two examples of this are:

Evaluate: $\sqrt{9}$

the square expression asks
"what positive number
times itself equals 9"

the answer is 3

Evaluate: $\sqrt[3]{64}$

the square expression asks
"what number as a product
three times itself equals 64"

the answer is 4

We will try to evaluate some basic logarithmic expressions without the use of algebraic steps by understanding what question the logarithmic expression is asking.

Evaluate: $\log_b a$

the log expression asks"
what power must b be
raised to to equal a"

or" what power of b equals a

Example 1

Evaluate: $\log_4 16$

the log expression asks"
what power must the base of 4
be raised to to equal 16"

The answer is 2 because $4^2 = 16$

so we know that $\log_4 16$ is 2

Example 2

Evaluate: $\log_2 32$

the log expression asks"
what power must the base of 2
be raised to to equal 32"

The answer is 5 because $2^5 = 32$

so we know that $\log_2 32$ is 5

Example 3

Evaluate: $\log_5 1$

the log expression asks"
what power must the base of 5
be raised to to equal 1"

The answer is 0 because $5^0 = 1$

so we know that $\log_5 1$ is 0

Example 4

Evaluate: $\log_6 6$

the log expression asks"
what power must the base of 6
be raised to to equal 6"

The answer is 1 because $6^1 = 6$

so we know that $\log_6 6$ is 1

Example 5

Evaluate: $\log 1000$

The log has a base of 10

the log expression asks"
what power must the base of 10
be raised to to equal 1000"

The answer is 3 because $10^2 = 1000$

so we know that $\log 1000$ is 3

Example 6

Evaluate: $\log .1$

The log has a base of 10

the log expression asks"
what power must the base of 10
be raised to to equal .1 or 1/10"

The answer is -1 because $10^{-1} = 1/10 = .1$

so we know that $\log .1$ is -1

Example 7

Evaluate: $\log_4 2$

the log expression asks"
what power must the base of 4
be raised to to equal 2"

The answer is 1/2 because $4^{1/2} = 2$

so we know that $\log_4 2$ is 1/3

Example 8

Evaluate: $\log_{125} 5$

the log expression asks"
what power must the base of 125
be raised to to equal 5"

The answer is 1/3 because $125^{1/3} = 3$

so we know that $\log_{125} 5$ is 1/3

Example 9

Evaluate: $\log_{(1/2)} 8$

the log expression asks"
what power must the base of $1/2$
be raised to to equal 8"

If you flip $1/2$ to get 2 and then
cube it you get 8

The answer is -3 because $\left(\frac{1}{2}\right)^{-3} = 8$

so we know that $\log_{(1/2)} 8$ is -3

Example 10

Evaluate: $\log_{(1/3)} 81$

the log expression asks"
what power must the base of $1/3$
be raised to to equal 81"

If you flip $1/3$ to get 3 and then
raise it to the forth power you get 64

The answer is -4 because $\left(\frac{1}{3}\right)^{-4} = 64$

so we know that $\log_{(1/3)} 81$ is -4

Example 11

Evaluate: $\log_4\left(\frac{1}{16}\right)$

the log expression asks"
what power must the base of 4
be raised to to equal $1/16$"

If you flip 4 to get $1/4$ and then
square it you get $1/16$

The answer is -2 because $(4)^{-2} = 1/16$

so we know that $\log_4\left(\frac{1}{16}\right)$ is -2

Example 12

Evaluate: $\ln e^4$

the base of $\ln$ is e

the log expression asks"
what power must the base of e
be raised to to equal e^4"

The answer is 4 because $e^4 = e^4$

so we know that $\ln e^4$ is 4

