

Section 8 – 4: Polynomial Inequalities in One Variable

Quadratic and Polynomial Inequalities in one variable have look like the example below.

$$x^2 - 5x - 6 \leq 0$$

$$(x - 2)(x + 4) > 0$$

$$x^2(x - 3) > 0$$

$$(x - 2)^2(x + 4) \geq 0$$

Most of the polynomials in this section are second degree and third degree polynomials. They are stated in terms of the variable x . Any of the four inequality symbols \geq , $>$, \leq or $<$ can be used to form the Inequality. It is also common to state the problem with the polynomial already isolated on one side of the inequality with zero on the other side. In many cases the polynomial will already be factored to avoid requiring the student to factor higher ordered polynomials.

Zero Factor Rule

The zero product rule says if you have several factors whose **product EQUAL zero** then **either the first factor is equal to zero or the second factor is equal to zero or etc.**

Roots or Critical Values

The roots or critical values of a polynomial are the x values where the polynomial **EQUALS** zero. To find the Roots of a Polynomial Inequality replace the inequity with and equal sign and solve the resulting equation.

Example 1

Find the Roots of
the Inequality

$$(x - 1)(x + 5) > 0$$

Solve the Equality

$$(x - 1)(x + 5) = 0$$

Set each factor equal
to 0 and solve for x

$$x - 1 = 0 \text{ or } x + 5 = 0$$

$$x = 1 \text{ or } x = -5$$

Roots: 1, -5

Example 2

Find the Roots of
the Inequality

$$2x(x + 3)^2(x - 4) \leq 0$$

Solve the Equality

$$2x(x + 3)^2(x - 4) = 0$$

Set each factor equal
to 0 and solve for x

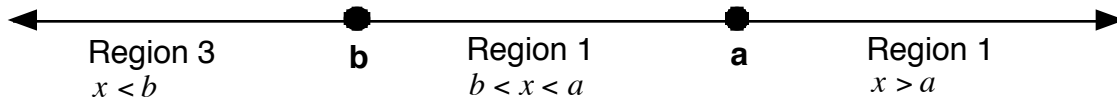
$$2x = 0 \text{ or } x + 3 = 0 \text{ or } x - 4 = 0$$

$$x = 0 \text{ or } x = -3 \text{ or } x = 4$$

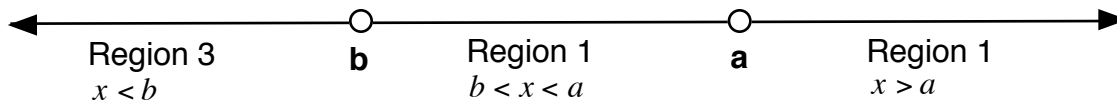
Roots: 0, -3, 4

If you **graph all the roots** (critical numbers) for the Polynomial **Equality** on a real number line, the line will be separated into several regions.

If the inequality is an \geq or \leq use a **closed circle** to graph each root.



If the inequality is an $>$ or $<$ use an **open circle** to graph each root.

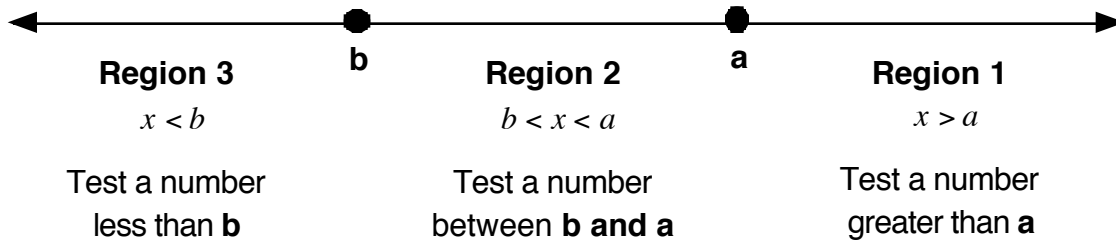


Each region contains an infinite number of x values. All of the x values in a regions either solve the Inequality or DO NOT Solve the Inequality. This says that if one x values in the region is a solution to the inequality, then all the values for x in that region are a solution. If one x values in the region is NOT a solution to the inequality, then all the values for x in that region are NOT a solution.

Test any one number you like in each of the regions.

**If the test number in that region is a solution to the inequality
than all the numbers in that region are a solution to the inequality.**

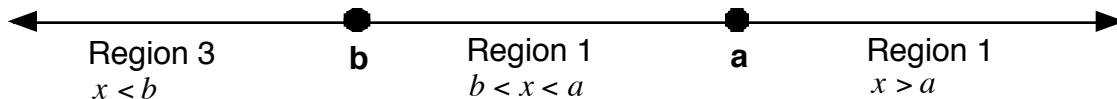
Shade the entire region to show all the numbers in that region are solutions.



Determine if the Roots are solutions

If the inequality is an \geq or \leq use a **closed circle** to graph each root.

Each root **IS** a solution the inequality



If the inequality is an $>$ or $<$ use an **open circle** to graph each root.

Each root is **NOT** a solution the inequality.

Graph the solution to each Inequality and then state the solution in interval notation.

Example 1: Solve $(x-1)(x+5) \geq 0$

Step 1. Find the roots

Solve the Equality

$$(x-1)(x+5) = 0$$

Set each factor equal

to 0 and solve for x

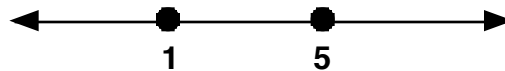
$$x-1=0 \text{ or } x+5=0$$

$$x=1 \text{ or } x=-5$$

Roots: 1, -5

Step 2. Graph each root on the number line

If the inequality is an \geq or \leq use a **closed circle** to graph each root.

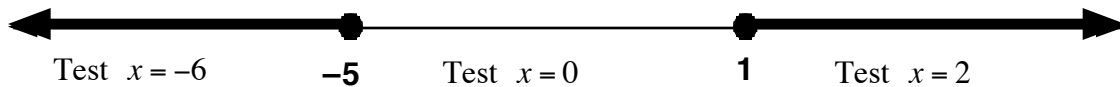


Test any one number you like in each of the 3 regions.

If that number is a solution in that region

than all the numbers in that region are a solution.

Shade the entire region



$$(x-1)(x+5) \geq 0$$

$$(-6-1)(-6+5)$$

$$(-7)(-1) \geq 0$$

$$7 \geq 0$$

-6 is a solution so

Shade this region

$$(x-1)(x+5) \geq 0$$

$$(0-1)(0+5)$$

$$(-1)(5) \geq 0$$

$$-5 \geq 0$$

0 is NOT a solution so

so DO NOT

Shade this region

$$(x-1)(x+5) \geq 0$$

$$(2-1)(2+5)$$

$$(1)(7) \geq 0$$

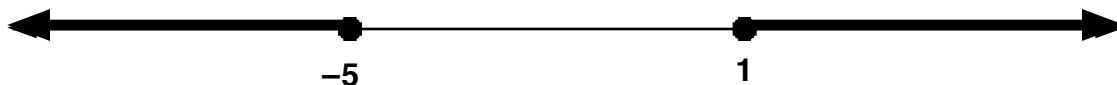
$$7 \geq 0$$

2 is a solution so

Shade this region

Solution:

$$\text{Solve } (x-1)(x+5) \geq 0$$



$$x \in \text{Reals} \mid x \in (-\infty , -5) \cup (1 , +\infty)$$

Example 2: Solve $2x(x - 7) < 0$

Step 1. Find the roots

Solve the Equality

$$2x(x - 7) = 0$$

Set each factor equal

to 0 and solve for x

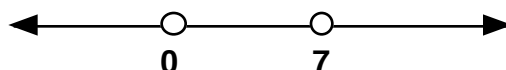
$$2x = 0 \text{ or } x - 7 = 0$$

$$x = 0 \text{ or } x = 7$$

Roots: 0 , 7

Step 2. Graph each root on the number line

If the inequality is an $<$ or $>$ use an **open circle** to graph each root.



Test any one number you like in each of the 3 regions.

If that number is a solution in that region

than all the numbers in that region are a solution.

Shade the entire region



Test $x = -1$

Test $x = 1$

Test $x = 8$

$$2x(x - 7) < 0$$

$$2(-1)(-1 - 7) < 0$$

$$2(-1)(-8) < 0$$

$$15 < 0$$

-2 is Not a solution

so DO NOT

Shade this region

$$(2x)(x - 7) < 0$$

$$2(1)(1 - 7) < 0$$

$$2(1)(-6) < 0$$

$$-12 < 0$$

1 is a solution so

Shade this region

$$2x(x - 7) < 0$$

$$2(8)(8 - 7) < 0$$

$$2(8)(1) < 0$$

$$16 < 0$$

8 is NOT a solution

so DO NOT

Shade this region

Solution:

$$\text{Solve: } 2x(x - 7) < 0$$



$$x \in \text{Reals} \mid x \in [0, 7]$$

Example 3: Solve $3x^2(x - 4) \geq 0$

Step 1. Find the roots

Solve the Equality

$$3x^2(x - 4) = 0$$

Set each factor equal
to 0 and solve for x

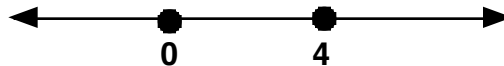
$$3x^2 = 0 \text{ or } x - 4 = 0$$

$$x = 0 \text{ or } x = 4$$

Roots: 0 , 4

Step 2. Graph each root on the number line

If the inequality is an \leq or \geq use a closed circle to graph each root.

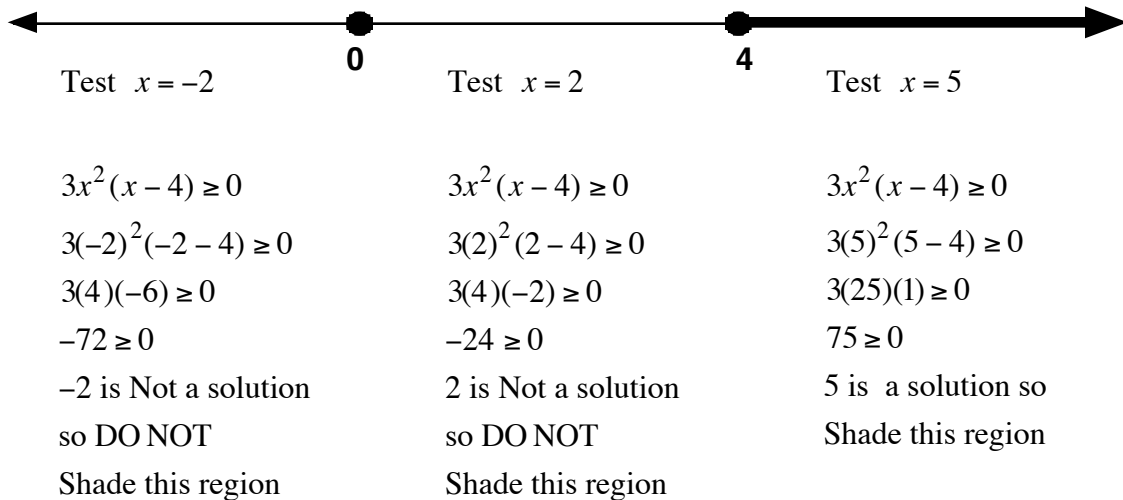


Test any one number you like in each of the 3 regions.

If that number is a solution in that region

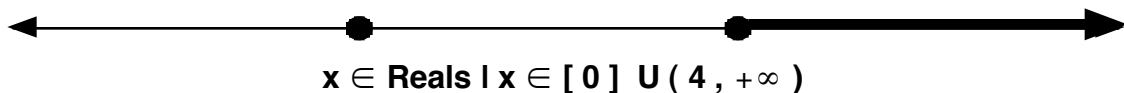
than all the numbers in that region are a solution.

Shade the entire region



Solution:

$$\text{Solve: } 3x^2(x - 4) \geq 0$$



Example 4: Solve $(x - 5)^2(x + 1) > 0$

Step 1. Find the roots

Solve the Equality

$$(x - 5)^2(x + 1) = 0$$

Set each factor equal
to 0 and solve for x

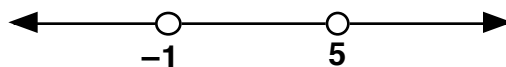
$$(x - 5)^2 = 0 \text{ or } x + 1 = 0$$

$$x = 5 \text{ or } x = -1$$

Roots: 5 , -1

Step 2. Graph each root on the number line

If the inequality is $<$ or $>$ use an open circle to graph each root.

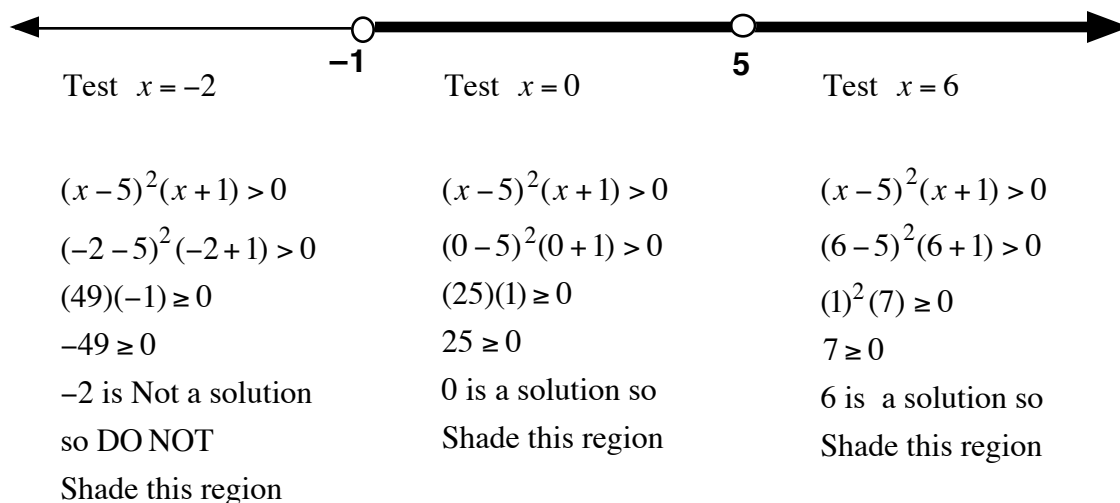


Test any one number you like in each of the 3 regions.

If that number is a solution in that region

than all the numbers in that region are a solution.

Shade the entire region



Solution:

Solve: $(x - 5)^2(x + 1) > 0$



$$x \in \text{Reals} \mid x \in (-1, 5) \cup (5, +\infty)$$

Example 5: Solve $(x + 4)^2(x - 3) \leq 0$

Step 1. Find the roots

Solve the Equality

$$(x + 4)^2(x - 3) = 0$$

Set each factor equal

to 0 and solve for x

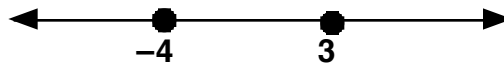
$$(x + 4)^2 = 0 \text{ or } x - 3 = 0$$

$$x = -4 \text{ or } x = 3$$

Roots: -4, 3

Step 2. Graph each root on the number line

If the inequality is an \leq or \geq use a closed circle to graph each root.



Test any one number you like in each of the 3 regions.

If that number is a solution in that region

than all the numbers in that region are a solution.

Shade the entire region



Test $x = -5$

-4

Test $x = 0$

3

Test $x = 5$

$$(x + 4)^2(x - 3) \leq 0$$

$$(-5 + 4)^2(-5 - 3) \leq 0$$

$$(-1)^2(-8) \leq 0$$

$$-8 \leq 0$$

-5 is a solution so

Shade this region

$$(x + 4)^2(x - 3) \leq 0$$

$$(0 + 4)^2(0 - 3) \leq 0$$

$$(4)^2(-3) \leq 0$$

$$-48 \leq 0$$

0 is a solution so

Shade this region

$$(5 + 4)^2(5 - 3) \leq 0$$

$$(5 + 4)^2(5 - 3) \leq 0$$

$$(9)^2(2) \leq 0$$

$$162 \leq 0$$

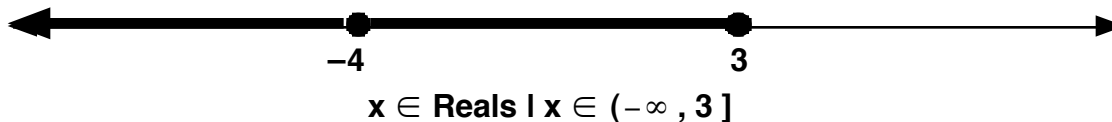
5 is NOT a solution

so DO NOT

Shade this region

Solution:

$$\text{Solve: } (x + 4)^2(x - 3) \leq 0$$



Example 6: Solve $(x)^2(x-4)(x+2)^2 > 0$

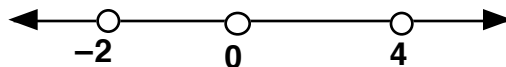
Solve the Equality

$$(x)^2(x-4)(x+2)^2 = 0$$

Set each factor equal to 0 and solve for x

Roots: 0, 4, -2

Step 2. Graph each root on the number line
The inequality is an $>$ so use a closed circle to graph each root.

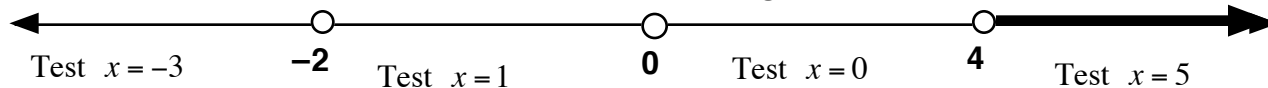


Test any one number you like in each of the 4 regions.

If that number is a solution in that region

than all the numbers in that region are a solution.

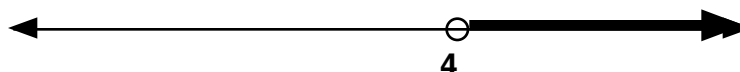
Shade the entire region



$(x)^2(x-4)(x+2)^2 > 0$ $(-3)^2(-3-4)(-3+2)^2 > 0$ $(9)(-7)(1) > 0$ $-63 > 0$ -3 Is Not a solution so DO NOT Shade this region	$(x)^2(x-4)(x+2)^2 > 0$ $(1)^2(1-4)(1+2)^2 > 0$ $(1)(-3)(9) > 0$ $-27 > 0$ 1 Is Not a solution so DO NOT Shade this region	$(x+4)^2(x-3) > 0$ $(0+4)^2(0-3) > 0$ $(4)^2(-3) > 0$ $-48 > 0$ 0 Is Not a solution so DO NOT Shade this region	$(x)^2(x-4)(x+2)^2 > 0$ $(5)^2(5-4)(5+2)^2 > 0$ $(25)(1)(49) > 0$ 5 Is a solution so Shade this region
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Solution:

Solve: $(x)^2(x-4)(x+2)^2 > 0$



$x \in \text{Reals} \mid x \in (4, +\infty)$

