Section 8 – 3: Applications and Formulas

Applications

Many word problems involve quadratic equations as part of the solution. The unit of measure for Area is a square unit. This means that area formulas involve quadratic equations. This section will introduce base area problems that are solved using quadratic equations.

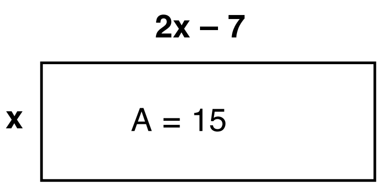
Example 1: Area of a Rectangle

The length of a rectangular flower bed is **7 less than twice the width**. The area of the flower bed is 15 square feet. Find the length and width.

Width = x

Length = 2x – 7

Area = 15



$A = x(2x-7)$

$15 = 2x^2 - 7x$

$2x^2 - 7x - 15 = 0$

$(2x+3)(x-5) = 0$

$x = \frac{-3}{2}$ or $x = 5$

Solution:

Length = 5 feet

Width = 3 Feet

Check $x = \frac{-3}{2}$

Width: $W = x = \frac{-3}{2}$

A negitive Width is NOT possiable

$x \neq \frac{-3}{2}$

Check $x = 5$

Width: $W = 5$

Width: $L = 2x - 7$

$L = 2(5)7 = 3$

$A = L \bullet W = 5 \bullet 3 = 15$

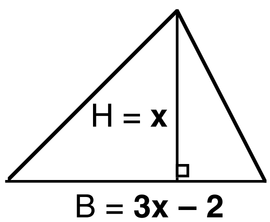
Example 2: Area of a Triangle

The base of a triangular sail is 2 less than three times the height. The area of the sail is 20 square feet. Find the base and height.

Height = x

Base = 3x – 2

Area = 20



$$A = \frac{1}{2} \bullet b \bullet h$$

$$20 = \frac{1}{2} \bullet (3x - 2) \bullet x$$

$$40 = 3x^2 - 2x$$

$$3x^2 - 2x - 40 = 0$$

$$(3x + 10)\ (x - 4) = 0$$

$$x = \frac{-10}{3} \quad \text{or} \quad x = 4$$

Solution:

Height = 4 feet

Base = 10 Feet

Check $x = \frac{-10}{3}$

Height: $H = x = \frac{-10}{3}$

A negitive Height is NOT possiable

$$x \neq \frac{-10}{3}$$

Check $x = 4$

Height: $H = x = 4$

Base: $B = 3x - 2$

$B = 3(4) - 2$

$B = 10$ \

$$A = \frac{1}{2} \bullet B \bullet H = \frac{1}{2} \bullet 10 \bullet 4 = 20$$

Formulas

Many formulas in Math and Science are stated in terms of 2 or more variables. A formula stated in terms of one variable like A in the equation $A = 4H^2$ can also be stated in terms of the other variable H. To do this you must solve for H. Notice that the H is a second degree term. This section will introduce the steps to solve for a variable that has a second degree exponent.

Steps used to solve a Quadratic Equation for a given variable

1. If there is a fraction in the equation multiply by the denominator to eliminate the fraction.

Solve for x

$H = \frac{1}{3} a \bullet x^2$ multiply both sides by 3

$3H = \cancel{3} \bullet \frac{1}{\cancel{3}} a \bullet x^2$

$3H = x^2$

Solve for m

$H = \frac{5m^2}{x}$ multiply both sides by x

$x \bullet H = \cancel{x} \bullet \frac{5m^2}{\cancel{x}}$

$xH = x^2$

2. If the term with the variable to be solved for has a separate term that has been added or subtracted from it then eliminate that separate term by adding or subtracting that term from both sides of the equation.

Solve for h

$4A \bullet = h^2 - 3w$ add 3w to both sides

$4A + 3w = h^2$

Solve for x

$P = x^2 + 5y$ subtract 5y form both sides

$P - 5y = x^2$

3. Divide (or multiply by the inverse) by the coefficient of the variable you wish to solve for.

Solve for h

$\frac{8b}{3} \bullet = a \bullet h^2$ multiply both sides by $\frac{1}{a}$

$\frac{1}{a} \bullet \frac{8b}{3} = \frac{1}{\cancel{a}} \bullet \cancel{a} \bullet h^2$

$\frac{8b}{3a} = h^2$

Solve for h

$4b = \frac{3}{2} a \bullet h^2$ multiply both sides by $\frac{2}{3}$

$\frac{2}{3} \bullet 4b = \frac{\cancel{2}}{\cancel{3}} \bullet \frac{\cancel{3}}{\cancel{2}} a \bullet h^2$

$\frac{8b}{3} \bullet = a \bullet h^2$

4. If the the variable you wish to solve for is alone but has an exponent then take the root of both sides of the equation to eliminate the exponent.

Solve for x

$3H = x^2$ take the square root of both sides

$\sqrt{3H} = x$

Solve for x

$\frac{2a}{b} = x^2$ take the square root of both sides

$\sqrt{\frac{2a}{b}} = x$

Solving a Quadratic Equation for a given variable

Example 1

Solve for x

$A = \frac{1}{2} b \bullet x^2$ multiply both sides by 2

$2A = 2 \bullet \frac{1}{2} b \bullet x^2$

$2A = b \bullet x^2$ divide both sides by b

$\frac{2A}{b} = \frac{\not{b} \bullet x^2}{\not{b}}$

$\frac{2A}{b} = x^2$ take the square root of both sides

$\sqrt{\frac{2A}{b}} = x$

Example 2

Solve for h

$4b = \frac{3}{2} a \bullet h^2$ multiply both sides by $\frac{2}{3}$

$\frac{2}{3} \bullet 4b = \frac{\not{2}}{\not{3}} \bullet \frac{\not{3}}{\not{2}} a \bullet h^2$

$\frac{8b}{3} \bullet = a \bullet h^2$ multiply both sides by $\frac{1}{a}$

$\frac{1}{a} \bullet \frac{8b}{3} = \frac{1}{\not{a}} \bullet \not{a} \bullet h^2$

$\frac{8b}{3a} = h^2$ take the square root of both sides

$\sqrt{\frac{8b}{3a}} = h$

Example 3

Solve for y

$P = x^2 + y^2$ subtract x^2 from both sides

$P - x^2 = x^2 + y^2 - x^2$

$P - x^2 = y^2$ take the square root of both sides

$\sqrt{P - x} = y$

Example 4

Solve for r

$A = 4\pi \bullet r^2$ divide both sides by 4π

$\frac{A}{4\pi} = r^2$ take the square root of both sides

$\sqrt{\frac{A}{4\pi}} = r$

