

Solving Quadratic Equations

Chapter 5 introduced the solution of second degree equations of the form $Ax^2 + Bx + C = 0$ by factoring and the use Zero Factor Rule. This is the simplest way to find the two solutions to a quadratic equation if the second degree equation can be factored.

The last section introduced the Quadratic Formula method to solve second degree equations of the form $Ax^2 + Bx + C = 0$ if the second degree expression **cannot be factored** or you do not want to solve by factoring. This method can be used to solve **ALL** Quadratic Equations but it requires more work than the factoring method.

If $B = 0$ then the Quadratic Equation $Ax^2 + Bx + C = 0$ **will have the form** $Ax^2 + C = 0$ Equations of the form $Ax^2 + C = 0$ can be solved by the Quadratic Equation method but there is an alternate method that may prove more desirable. This method is based on the Principle of Square roots.

The Principle of Square Roots

Solve: $x^2 = 25$

We can see by substitution
that both $x = 5$ and $x = -5$
are both solutions to the equation
since

$$(5)^2 = 25 \text{ and } (-5)^2 = 25$$

so we can say that the two solutions are

$$x = \pm 5$$

Solve: $x^2 = 7$

We can see by substitution
that both $x = \sqrt{7}$ and $x = -\sqrt{7}$
are both solutions to the equation
since

$$(\sqrt{7})^2 = 7 \text{ and } (-\sqrt{7})^2 = 7$$

so we can say that the two solutions are

$$x = \pm \sqrt{7}$$

The Principle of Square Roots

If

$$x^2 = K \text{ then}$$

$$\sqrt{x^2} = \pm \sqrt{K}$$

or

$$x = \sqrt{K} \text{ or } x = -\sqrt{K}$$

Solving Quadratic Equations of the form $Ax^2 + C = 0$

Step 1: Solve $Ax^2 + C = 0$ for the x^2 quantity.

Step 2. Use the Principle of Square Roots to solve for x

Example 1

Solve: $x^2 - 7 = 0$

Solve for x^2

$$x^2 = 7$$

take the square root of both sides
of the equation

$$\sqrt{x^2} = \sqrt{7}$$

the $\sqrt{x^2}$ requires the use of \pm

$$x = \pm \sqrt{7}$$

$$x = +\sqrt{7} \text{ or } x = -\sqrt{7}$$

Check: $x = \sqrt{7}$

Check: $x = -\sqrt{7}$

$$(\sqrt{7})^2 = 7$$

$$(-\sqrt{7})^2 = 7$$

$$7 = 7$$

$$7 = 7$$

Example 2

Solve: $4x^2 - 3 = 0$

Solve for x^2

$$x^2 = \frac{3}{4}$$

take the square root of both sides
of the equation

$$\sqrt{x^2} = \frac{\sqrt{3}}{\sqrt{4}}$$

the $\sqrt{x^2}$ requires the use of \pm

$$x = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$$

$$x = +\frac{\sqrt{3}}{2} \text{ or } x = -\frac{\sqrt{3}}{2}$$

Check: $x = \frac{\sqrt{3}}{2}$

Check: $x = -\frac{\sqrt{3}}{2}$

$$4\left(\frac{\sqrt{3}}{2}\right)^2 - 3 = 0$$

$$4\left(\frac{-\sqrt{3}}{2}\right)^2 - 3 = 0$$

$$4\left(\frac{3}{4}\right) - 3 = 0$$

$$4\left(\frac{3}{4}\right) - 3 = 0$$

$$3 - 3 = 0$$

$$3 - 3 = 0$$

Example 3

Example 4

$$\text{Solve: } x^2 + 16 = 0$$

Solve for x^2

$$x^2 = -16$$

take the square root of both sides
of the equation

$$\sqrt{x^2} = \sqrt{-16}$$

the $\sqrt{x^2}$ requires the use of \pm

$$x = \pm 4i$$

$$x = 4i \text{ or } x = -4i$$

Check: $x = 4i$ Check: $x = -4i$

$$x^2 + 16 = 0$$

$$(4i)^2 + 16 = 0 \quad (-4i)^2 + 16 = 0$$

$$16i^2 + 16 = 0 \quad 16i^2 + 16 = 0$$

$$-16 + 16 = 0 \quad -16 + 16 = 0$$

Example 5

$$\text{Solve: } x^2 + 5 = 0$$

Solve for x^2

$$x^2 = -5$$

take the square root of both sides
of the equation

$$\sqrt{x^2} = \sqrt{-5}$$

the $\sqrt{x^2}$ requires the use of \pm

$$x = \pm i\sqrt{5}$$

$$x = i\sqrt{5} \text{ or } x = -i\sqrt{5}$$

Check: $x = 4i$ Check: $x = -4i$

$$x^2 + 5 = 0$$

$$(i\sqrt{5})^2 + 5 = 0 \quad (-i\sqrt{5})^2 + 5 = 0$$

$$5i^2 + 16 = 0 \quad 5i^2 + 5 = 0$$

$$-5 + 5 = 0 \quad -5 + 5 = 0$$

Example 6

Solve: $(x - 3)^2 - 5 = 0$

Solve for $(x + 3)^2$

$$(x - 3)^2 = 5$$

take the square root of both sides
of the equation

$$\sqrt{(x - 3)^2} = \sqrt{5}$$

the $\sqrt{(x - 3)^2}$ requires the use of \pm

$$x - 3 = \pm \sqrt{5}$$

solve for x

$$x = 3 + \sqrt{5} \text{ or } x = 3 - \sqrt{5}$$

Solve: $(x + 4)^2 - 12 = 0$

Solve for $(x + 4)^2$

$$(x + 4)^2 = 12$$

take the square root of both sides
of the equation

$$\sqrt{(x + 4)^2} = \sqrt{12}$$

the $\sqrt{(x + 4)^2}$ requires the use of \pm

$$x + 4 = \pm \sqrt{12}$$

$$x + 4 = \pm 2\sqrt{3}$$

solve for x

$$x = -4 + 2\sqrt{3} \text{ or } x = -4 - 2\sqrt{3}$$

Example 7

Example 8

Solve: $(2x - 5)^2 - 7 = 0$

Solve for $(2x - 5)^2$

$$(2x - 5)^2 = 7$$

take the square root of both sides
of the equation

$$\sqrt{(2x - 5)^2} = \sqrt{7}$$

the $\sqrt{(2x - 5)^2}$ requires the use of \pm

$$2x - 5 = \pm \sqrt{7}$$

solve for x

$$x = \frac{5 \pm \sqrt{7}}{2}$$

Solve: $(3x + 7)^2 + 16 = 0$

Solve for $(3x + 7)^2$

$$(3x + 7)^2 = -16$$

take the square root of both sides
of the equation

$$\sqrt{(3x + 7)^2} = \sqrt{-16}$$

the $\sqrt{(3x + 7)^2}$ requires the use of \pm

$$3x + 7 = \pm 4i$$

solve for x

$$x = \frac{-7 \pm 4i}{3}$$