Section 7 – 5: Solving Radical Equations

Radical Equations are equations that have a radical expression in one or more of the terms in the equation. Most of the radicals equations in the section will involve square roots.

Example 1

$\sqrt{x-3} = 5$

Example 2

$\sqrt{x-3} + 2 = 5$

Example 3

$\sqrt{x-3} = x-2$

Other roots (cube , fourth etc.) could also be used.

Example 4

$\sqrt[3]{x-1} = 3$

Example 5

$\sqrt[4]{x+4} = 2$

The term with a square root must be eliminated to solve the equation. Squaring a square root expression gives an expression without a radical sign. Raising a cube root expression to the third power gives an expression without a radical sign.

Example 4

$\left(\sqrt{3x}\right)^2$

$= 3x$

Example 5

$\left(\sqrt{2x-3}\right)^2$

$= 2x-3$

Example 5

$\left(\sqrt[3]{4x-1}\right)^3$

$= 4x-1$

How to Solve Radical Equations

1. Get the term with the radical sign **alone one one side of the equation.**

2A. If the index of the radical is an even number (a square root or fourth root etc.) then it **CANNOT equal a negative number.** If the square root is alone is equal to a negative number then **STOP**. There are No Real Numbers that will work. We write NRN as the answer. We have solved the equation and **do not go on.**

2B. If the index of the radical is an odd number (a cube root or fifth root etc.) then it **CAN equal a negative number. Proceed to the next step.**

3. If the radical term is a square root then square both sides of the equation. If the radical term is a cube root then cube both sides. In general, raise each side of the equation to the appropriate power to eliminate the radical sign. This step will give an equation without a square root term in it.

4. Solve for x.

5. **Check your answer(s)**. If no answers work the there are No Real Numbers that are a solution. This step is critical. Squaring both sides can cause an value for x to be found that is NOT a solution to the original equation.

How to Solve Radical Equations Examples

Example 1

Solve for x

$\sqrt{2x}+8=14$

Get the square root alone
on one side of the equation
by subtracting 8 from both sides

$\sqrt{2x}=6$

square both sides of the equation

$\left(\sqrt{2x}\right)^2=6^2$

$2x=36$ solve for x

$x=18$

Check:

$\sqrt{2(18)}+8=14$

$\sqrt{36}+8=14$

$6+8=14$

Solution: $x=18$

Example 2

Solve for x

$\sqrt{8x}-5=-1$

Get the square root alone
on one side of the equation
by adding 5 to both sides

$\sqrt{8x}=4$

square both sides of the equation

$\left(\sqrt{8x}\right)^2=4^2$

$8x=16$ solve for x

$x=2$

Check:

$\sqrt{8(2)}-5=-1$

$\sqrt{16}-5=-1$

$4-5=-1$

Solution: $x=2$

Example 3

Solve for x

$\sqrt{2x}+7=3$

Get the square root alone
on one side of the equation
by subtracting 7 from both sides

$\sqrt{2x}=-4$

STOP: An isolated square root
cannot equal a negitive number
so the answer is NRN

Had you continued you would
have found that
$x=8$
but that answer would not have
worked if you checked it by
putting it back into the equation.

Solution: No Solution or $\varnothing$
or NRN

Example 4

Solve for x

$\sqrt{3x-5} = \sqrt{5x-9}$
square both sides

$\left(\sqrt{3x-5}\right)^2 = \left(\sqrt{5x-9}\right)^2$

$3x-5 = 5x-9$ solve for x

$4 = 2x$

$2 = x$

Check:
$\sqrt{3(2)-5} = \sqrt{5(2)-9}$
$\sqrt{6-5} = \sqrt{5(2)-9}$
$\sqrt{1} = \sqrt{1}$

Solution: x = 3

Example 5

Solve for x

$\sqrt{6x-1} = \sqrt{2x+11}$
square both sides

$\left(\sqrt{6x-1}\right)^2 = \left(\sqrt{2x+11}\right)^2$

$6x-1 = 2x+11$ solve for x

$4x = 12$

$x = 3$

Check:
$\sqrt{6(3)-1} = \sqrt{2(3)-9}$
$\sqrt{18-1} = \sqrt{2(3)+11}$
$\sqrt{17} = \sqrt{17}$

Solution: x = 3

Example 6

Solve for x

$\sqrt{5x+2} = \sqrt{2x-7}$
square both sides

$\left(\sqrt{5x+2}\right)^2 = \left(\sqrt{2x-7}\right)^2$

$5x+2 = 2x-7$ solve for x
$3x = -9$
$x = -3$

Check:
$\sqrt{5(-3)+2} = \sqrt{2(-3)-7}$
$\sqrt{-15+2} = \sqrt{-6-7}$
$\sqrt{-13} = \sqrt{-13}$
We cannot have the Square Root of a negitive so NRN

Solution: No Solution or ∅ or NRN

Example 7

Solve for x

$\sqrt{3x+10} = x$
square both sides

$\left(\sqrt{3x+10}\right)^2 = (x)^2$

$3x+10 = x^2$
Now solve for x

$0 = x^2 - 3x - 10$

$0 = (x-5)\ (x+2)$

$x = 5$ or $x = -2$

Check: x = 5
$\sqrt{3(5)+10} = 5$ yes

Check: x = -2
$\sqrt{3(-2)+10} = -2$ No

Solution: x = 5

Example 8

Solve for x

$\sqrt{3x-2} = x-4$
square both sides

$\left(\sqrt{3x-2}\right)^2 = (x-4)^2$

$3x-2 = x^2 - 8x + 16$
Now solve for x

$0 = x^2 - 11x + 18$

$0 = (x-9)\ (x-2)$

$x = 9$ or $x = 2$

Check: x = 9
$\sqrt{3(9)-2} = 9-4$
$\sqrt{27-2} = 5$ Yes

Check: x = 2
$\sqrt{3(2)-2} = 2-4$
$\sqrt{6-2} = -2$ No

Solution: x = 9

Example 9

Solve for x

$\sqrt{3x+4} = 2x+1$
square both sides

$\left(\sqrt{3x+4}\right)^2 = (2x+1)^2$

$3x+4 = 4x^2 + 4x + 1$
Now solve for x

$0 = 4x^2 + x - 3$

$0 = (4x-3)\ (x+1)$

$x = 3/4$ or $x = -1$

Check: x = 3 / 4
$\sqrt{3\left(\frac{3}{4}\right)+4} = 2\left(\frac{3}{4}\right)+1$ Yes

Check: x = −1
$\sqrt{3(-1)+4} = 2(-1)+1$ No

Solutions: $x = \frac{3}{4}$

How to Solve Radical Equations with Cube Roots

1. Get the term with the cube root **alone one one side of the equation.**
2. Cube both sides.
4. Solve for x
5. Check your answer(s). If no answers work the there is No Real Number that is a solution.

Example 10

Solve for x

$\sqrt[3]{2x}+8=10$

Get the cube root alone
on one side of the equation
by subtracting 8 from both sides

$\sqrt[3]{2x}=2$

cube both sides of the equation

$\left(\sqrt[3]{2x}\right)^3=2^3$

$2x=8$ solve for x

$x=4$

Check: $x=4$

$\sqrt[3]{2(4)}+8=10$

$\sqrt[3]{8)}+8=10$

$2+8=10$ Yes

Solution: $x=4$

Example 11

Solve for x

$\sqrt[3]{x-2}+1=4$

Get the cube root alone
on one side of the equation
by subtracting 1 from both sides

$\sqrt[3]{x-2}=3$

cube both sides of the equation

$\left(\sqrt[3]{x-2}\right)^3=3^3$

$x-2=27$ solve for x

$x=29$

Check: $x=29$

$\sqrt[3]{29-2}+1=4$

$\sqrt[3]{27}+1=4$

$3+1=4$ Yes

Solution: $x=29$

Example 12

Solve for x

$\sqrt[3]{x+3}+5=4$

Get the cube root alone
on one side of the equation
by subtracting 5 from both sides

$\sqrt[3]{x+3}=-1$

cube both sides of the equation

$\left(\sqrt[3]{x+3}\right)^3=(-1)^3$

$x+3=-1$ solve for x

$x=-4$

Check: $x-4$

$\sqrt[3]{-4+3}+5=4$

$\sqrt[3]{-1}+5=4$

$-1+5=4$ Yes

Solution: $x=-4$

